UG–467 **BMS-04**

B.Sc. DEGREE EXAMINATION
JANUARY 2009.

(AY – 2005-06 and CY – 2006 batches only)

Second Year

Mathematics

MODERN ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. If the Function $f : R \to R$ is given by $f(x) = x^2$ and $g : R \to R$ is given by $g(x) = \sin x$, find $(g \circ f)(x)$ and $(f \circ g)(x)$ and show that they are not equal.

2. If H and K are subgroups of a given group G, then prove that $H \cap K$ is also a subgroup of G.

3. How many generators are there for a cyclic group of order 10?

4. If n is any integer and $(a,n)=1$ then prove that $a^{\phi(n)} \equiv 1 \pmod{n}$.

5. Prove that every subgroup of an abelian group is a normal subgroup.

6. If $f : G \to G'$ is a homomorphism then show that f is one to one if and only if $\ker f = \{e\}$.

7. Prove that the set of all matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ where $a, b \in R$ is a ring under matrix addition and multiplication.

8. If R is a commutative ring with identity then prove that R is an integral domain if and only if cancellation law is valid in R.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If A and B are two subgroups of a group G then prove that AB is a subgroup of G if and only if $AB = BA$.

10. If G is a group and $a \in G$ then prove that the order of a is the same as the order of the cyclic group generated by a.

11. Prove that a group G has no proper subgroups if and only if it is a cyclic of prime order.

12. If a group G has exactly one subgroup H of given order then prove that H is a normal subgroup of G.

13. Prove that any finite group is isomorphic to a group of permutations.

14. State and prove the fundamental theorem of homomorphism.

15. Prove that any finite integral domain is a field.

16. Prove that any integral domain D can be embedded in a field F and every element of F can be expressed as a quotient of two elements of D.