UG–471 **BMS–08**

B.Sc. DEGREE EXAMINATION –
JANUARY 2009.

Third Year

(A.Y. 2005–06 and C.Y. 2006 batches only)

Mathematics

LINEAR ALGEBRA AND NUMBER SYSTEM

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that the set of complex numbers C is a vector space over the field **R**.

2. Define inner product space. Give an example.

3. Compute the inverse of the matrix $\begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$.

4. Verify Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & 2 \\ 3 & -1 & 3 \end{pmatrix}$.

5. Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{pmatrix}$.

6. Find the smallest number with 18 divisors.

7. Show that $x^5 - x$ is divisible by 30.

8. Show that $7^{2n} + 16n - 1 \equiv 0 \pmod{64}$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let V be a vector space over a field F. Let $S = \{v_1, v_2, \cdots, v_n\} \subseteq V$. Prove that the following are equivalent

(a) S is a basis for V.

(b) S is a Maximal linear independent set

(c) S is a minimal generating set.

10. If V is a finite dimensional vector space over a field F and W is a subspace of V, prove that $\dim \frac{V}{W} = \dim V - \dim W$.

11. Apply Gram-Schmidt process to construct orthonormal basis from the basis {(1, 0, 1), (1, 3, 1), (3, 2, 1)}.

12. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.

13. Reduce the matrix $\begin{pmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{pmatrix}$ to its normal form.

14. Define Eulers ϕ-function. Find the value $\phi(N)$, if $N = p^a q^b r^c \cdots$ where $p, q, r, \cdots$ are all primes and $a, b, c, \cdots$ are integers.

15. State and prove Fermat's theorem.

16. Show that $28! + 233 \equiv 0 \pmod{899}$.
