

UG-476

**BMS-12/
BMC-12**

**B.Sc. DEGREE EXAMINATION —
JANUARY, 2009.**

First Year

(AY 2006-07 batch onwards)

Mathematics

**TRIGONOMETRY, ANALYTICAL GEOMETRY OF
THREE DIMENSIONS AND VECTOR CALCULUS**

Time : 3 hours

Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. Prove that $\frac{\sin 5\theta}{\sin \theta} = 16 \sin^4 \theta - 20 \sin^2 \theta + 5$.
2. Prove that $1 - \tanh^2 x = \operatorname{sech}^2 x$.
3. If $\frac{\tan \theta}{\theta} = \frac{2524}{2523}$, find θ approximately.

4. Find the equation to the plane through $(2,-4,5)$ and is parallel to the plane $4x + 2y - 7z + 6 = 0$.
5. Find the straight line through $(3,2,-8)$ and perpendicular to $-3x + y + 2z - 2 = 0$.
6. Find the centre and radius of the sphere $7x^2 + 7y^2 + 7z^2 + 28x - 42y + 56z + 3 = 0$.
7. Find unit vector normal to the surface $x^2 + 2y^2 + z^2 = 7$ at $(1,-1,2)$
8. Prove that $\text{curl}(\text{grad}\phi) = \vec{0}$.

PART B — $(5 \times 10 = 50$ marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. Prove that $2^5 \cos^6 \theta = \cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10$.
10. If $\cos(x + iy) = \cos a + i \sin a$ prove that $\cos 2x + \cosh 2y = 2$.
11. Sum the series $\sin \theta + \frac{\sin 2\theta}{2} + \frac{\sin 3\theta}{3} + \dots \infty$.

12. Find the equation of the plane through (2,-3,1) and is perpendicular to the line joining the points (3,4,-1) and (2,-1,5).

13. Show that the lines $\frac{x+3}{2} = \frac{y+5}{3} = \frac{z-7}{-3}$ and $\frac{x+1}{4} = \frac{y+1}{5} = \frac{z+1}{-1}$ are coplanar and find the equation of the plane containing them.

14. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1,2,3).

15. Show that $\vec{F} = yz\hat{i} + 3x\hat{j} + xy\hat{k}$ is irrotational. Find ϕ so that $\vec{F} = \nabla\phi$.

16. Show that Green's theorem in a plane can be deduced as a special case of Stoke's theorem.

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