UG–478 **BMS–21**

B.Sc. DEGREE EXAMINATION –
JANUARY 2009.

Second Year

Mathematics

GROUPS AND RINGS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Show that $f : R \to R$ defined by $f(x) = 2x - 3$ is a bijection and find its inverse. Compute $f^{-1} \circ f$ and $f \circ f^{-1}$.

2. State and prove second principle of induction.

3. If A_n is the set of all even permutations in S_n, then prove that A_n is a group containing $\frac{n!}{2}$ permutations.

4. Prove that a subgroup of a cyclic group is cyclic.

5. If the index of a subgroup H of a group G is two, then show that $aH = Ha$, for every $a \in G$.

6. If G is any group and $a \in G$, then show that $\phi_a : G \to G$ defined by $\phi_a(x) = axa^{-1}$ is an automorphism of G.

7. Prove that any finite integral domain is a field.

8. Prove that any Euclidean domain R has an identity element.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. If $f : A \to B$ and $g : B \to C$ are bijections, then show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

10. If A and B are two subgroups of a group G, then prove that AB is a subgroup of G if and only if $AB = BA$.

11. If G is a group and $a, b \in G$, then show that

(a) order of a = order of a^{-1}

(b) order of a = order of $b^{-1}ab$

(c) order of ab = order of ba.

12. State and prove Lagrange's theorem. Discuss about its converse.

13. If G is a cyclic group generated by a and $f: G \to G$ is a mapping such that $f(xy) = f(x) f(y)$, then prove that f is an automorphism of G if and only if $f(a)$ is a generator of G.

14. If R is a commutative ring with identity, then prove that every maximal ideal of R is prime ideal of R.

15. Prove that for any prime p, Z_p is not an ordered integral domain.

16. Show that the ring of Gaussian integers is an Euclidean domain.