UG-470

BMS-07

B.Sc. DEGREE EXAMINATION -JANUARY 2009.

(AY - 2005-06 and CY - 2006 batches only)

Third Year

Mathematics

REAL AND COMPLEX ANALYSIS

Time : 3 hours

m.com Maximum marks: 75

PART A — $(5 \times 5 = 25 \text{ marks})$

Answer any FIVE questions.

Each question carries 5 marks.

1. Prove that in any metric space, the union of any family of open sets is open.

 $\mathbf{2}$. Define a convergent sequence. Show that for a convergent sequence \mathbf{e}_n the limit is unique.

3. If (M, d) is a metric space and $a \in M$, prove that the functions $f: M \to R$ defined by $f \notin = d \notin a$ is continuous.

4. Show that any discrete metric space with more than one point is disconnected.

Show that the function $f \oint = \frac{\overline{z}}{z}$ does not have a 5. limit as $z \to 0$.

6. Find the bilinear transformation which maps the points $z = -1, 1, \infty$ respectively on w = -i, -1, i.

7. State and prove Lioville's theorem.

Nam. com Define residue of $f \notin at$ an isolated singularity 8. calculate the residues of $\frac{z+1}{z^2-2z}$ at its poles.

PART B — $(5 \times 10 = 50 \text{ marks})$

Answer any FIVE questions.

Each question carries 10 marks.

State and prove Minkowski's inequality. 9.

10. Show that l_2 is complete.

Prove that f is continuous if and only if inverse 11. image of every open set is open.

Show that any compact subset A of a metric space 12.(M, d) is closed.

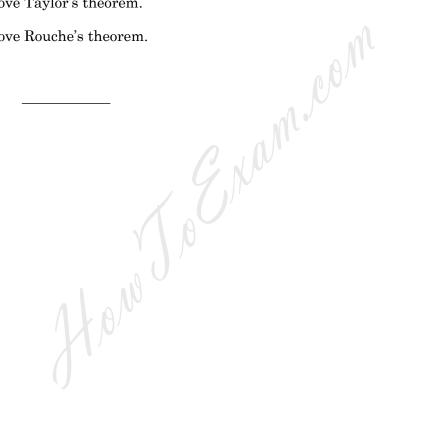
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Show that the points z_1 and z_2 are reflection 13.points for the line $\overline{\alpha}z + \alpha \overline{z} + \beta = 0$ if and only if $\overline{\alpha} z_1 + \alpha \overline{z}_2 + \beta = 0.$

State and prove a sufficient condition for 14. differentiability of complex valued function.

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- State and prove Taylor's theorem. 15.
- 16. State and prove Rouche's theorem.



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