UG–470 **BMS–07**

B.Sc. DEGREE EXAMINATION –
JANUARY 2009.

(AY – 2005-06 and CY – 2006 batches only)

Third Year

Mathematics

REAL AND COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. Prove that in any metric space, the union of any family of open sets is open.

2. Define a convergent sequence. Show that for a convergent sequence (x_n) the limit is unique.

3. If (M, d) is a metric space and $a \in M$, prove that the functions $f : M \to R$ defined by $f(x) = d(x, a)$ is continuous.

4. Show that any discrete metric space with more than one point is disconnected.

5. Show that the function $f(z) = \frac{\bar{z}}{z}$ does not have a limit as $z \to 0$.

6. Find the bilinear transformation which maps the points $z = -1, 1, \infty$ respectively on $w = -i, -1, i$.

7. State and prove Lioville's theorem.

8. Define residue of $f(z)$ at an isolated singularity calculate the residues of $\frac{z+1}{z^2 - 2z}$ at its poles.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. State and prove Minkowski's inequality.

10. Show that l_2 is complete.

11. Prove that f is continuous if and only if inverse image of every open set is open.

12. Show that any compact subset A of a metric space (M, d) is closed.

13. Show that the points z_1 and z_2 are reflection points for the line $\bar{\alpha}z + \alpha\bar{z} + \beta = 0$ if and only if $\bar{\alpha}z_1 + \alpha\bar{z}_2 + \beta = 0$.

14. State and prove a sufficient condition for differentiability of complex valued function.

15. State and prove Taylor's theorem.

16. State and prove Rouche's theorem.
