

B.E. I YEAR MAIN EXAMINATION, JUNE 2010
MATHEMATICS - I

Time: 3 Hours]

[Max. Marks: 75

Note: Answer **all** questions from **Part A**.
Answer **any** five questions from **Part B**.

PART – A [MARKS 25]

1. Are the vectors $(1, 0, 1)$, $(0, 1, 1)$, $(2, 2, 4)$ linearly dependent? If so, find the relation between them. [MARKS 3]

2. Find the sum of the eigen values of the matrix $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 5 & 6 \\ 1 & 2 & 4 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$ [MARKS 2]

3. Test the series $\sum(-1)^{n-1}(1/n2^n)$ for absolute convergence. [MARKS 3]

4. Discuss the convergence of the series $\sum \frac{n^2+1}{n^2}$. [MARKS 2]

5. Expand $f(x) = \tan x$ about $x = 0$ upto the term containing x^3 . [MARKS 3]

6. Find the radius of curvature of $x^2 + y^2 = 4$ at $(1, 2)$. [MARKS 2]

7. Determine $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+y^2}{x-y}$, if it exists. [MARKS 2]

8. If $z = e^{ax+by}f(ax-by)$, then pprove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. [MARKS 3]

9. Evaluate $\iint xy \, dx \, dy$ over the first quadrant of the circle $x^2 + y^2 = 4$. [MARKS 3]

10. Find the directional derivative of $f(x,y) = x^2 + y^2$ at $(1, 1)$ in the direction of $2i-4j$. [MARKS 2]

PART – B [MARKS 50]

11. a) Using Caley-Hamilton theorem, find the inverse of matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

[MARKS 5]

b) Reduce the quadratic form $Q = x^2 + 3y^2 + 3z^2 - 2yz$ to canonical form [MARKS 5]

12. a) Discuss the convergence of the geometric series $\sum r^n$, r is any real number. [MARKS 5]

b) Test the series $\sum(\sqrt{(n^4+1)} - \sqrt{(n^4-1)})$ for convergence. [MARKS 5]

13. a) State and prove Cauchy's mean value theorem. [MARKS 5]

b) Find the envelop of $\frac{x}{a} \cos \alpha + \frac{y}{b} \sin \alpha = 1$, α is a parameter. [MARKS 5]

14. a) Sketch the graph of the curve $x^2 - xy + 1 = 0$. [MARKS 5]

b) Find the local minimum and local maximum values of the function

$$f(x, y) = x^3 + y^3 - 3xy. \quad [\text{MARKS } 5]$$

15. a) State and prove Green's theorem in a plane. [MARKS 5]

b) If \vec{a} is a constant vector and $\vec{r} = xi + yj + zk$, find $\text{curl}(\vec{a} \times \vec{r})$. [MARKS 5]

16. a) Reduce the matrix $\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ to normal form and hence find its rank. [MARKS 5]

b) Discuss the convergence of the series $\sum((n + 1)^4/(n^{n+1}))x^n$, $x > 0$. [MARKS 5]

17. a) If $z = f(x, y)$, $x = e^{2u} + e^{-2v}$, $y = e^{-2u} + e^{2v}$, then show that $\frac{\partial f}{\partial u} - \frac{\partial f}{\partial v} = 2(x\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y})$. [MARKS 5]

b) Evaluate $\iint_S \vec{F} \cdot \vec{n} \, ds$, where $\vec{F} = x^2i + 3y^2k$ and S is the position of the plane $x + y + z = 1$ in the first octant. [MARKS 5]