

FACULTY OF ENGINEERING

B.E. II/IV Year (ECE/Mech/Prod/CSE) II Semester (Supplementary)
Examination, December 2008

MATHEMATICS — IV

Time : 3 Hours]

[Max. Marks : 75

Answer **all** questions of Part A.
Answer any **five** questions from Part B.

Part A – (Marks : 25)

Choose the correct answer from the following.

1. If $f(z) = x + ay + i (bx + cy)$ is analytic the values of $a = \text{---}$ and $c = \text{---}$.
(a) $a = b, c = 1$ (b) $a = c, b = 1$ (c) $a = -1, b = 1$ (d) $a = -b, c = 1$
2. The value of $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x^2$ is
(a) $\frac{5-i}{6}$ (b) $\frac{5+i}{6}$ (c) $\frac{i-5}{6}$ (d) None
3. The value of $\int_C \frac{z^3 - z}{(z-2)^3} dz$ where C is $|z| = 3$, using Cauchy's Integral formula is
(a) $12 \pi i$ (b) $-12 \pi i$ (c) πi (d) None
4. Residue of $f(z) = \frac{1 - e^{2x}}{z^4}$ at $z = 0$ is
(a) $\frac{8}{3}$ (b) $\frac{-4}{3}$ (c) $\frac{4}{3}$ (d) None
5. Using Cauchy's Residue theorem the value of $\int_C \frac{(z-1) dz}{(z+1)^2 (z-2)}$ where C is the circle $|z-i| = 2$ is
(a) $\frac{\pi i}{9}$ (b) $\frac{-\pi i}{9}$ (c) $\frac{-2\pi i}{9}$ (d) None
6. The Mean of Poisson distribution (λ as a parameter) is --- .
(a) λ^2 (b) λ (c) 2λ (d) None

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7. If x and y are random variables then $v(x - y)$ is _____.
- (a) $v(x) + v(y)$ (b) $v(x) - v(y)$
 (c) $v(x) + v(y) + 2 \text{cov}(x, y)$ (d) $v(x) + v(y) - 2 \text{cov}(x, y)$
8. Correlation coefficient is independent of change of scale and change of origin. (TRUE/FALSE)
9. Find the correlation coefficient (r) for the following data:
- | | | | | | | | |
|-----|---|---|---|---|---|---|---|
| x | : | 1 | 2 | 3 | 4 | 5 | 6 |
| y | : | 6 | 5 | 4 | 3 | 2 | 1 |
10. Derive Mean and Variance for the normal distribution.

Part B - (Marks : 50)

11. (a) If $f(z) = u + iv$ is analytic, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) | \text{Re } f(z) |^2 = 2 | f'(z) |^2$.
- (b) Find the bilinear transformation that maps $z = 0, -i, -1$ onto $w = i, 1, 0$.
12. (a) Using Cauchy's integral formula evaluate $\int_C \frac{z^3 - z dz}{(z-2)^3}$ where C is $|z-2| = 1$.
- (b) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ $a > 0, b > 0$ using residue theorem.
13. (a) If X is a random variable with the following distribution
- | | | | | |
|--------|-----|-----|-----|-----|
| x | 1 | 3 | 4 | 5 |
| $f(x)$ | 0.4 | 0.1 | 0.2 | 0.3 |
- Find the mean, variance and standard deviation of X .
- (b) A pair of dice is thrown. Let X denote the minimum of the two numbers which occur. Find the distribution and expectation of X .
14. (a) Let X be normally distributed with mean $\mu = 8$ and standard deviation $\sigma = 4$. Find (i) $P(5 \leq X \leq 10)$, (ii) $P(10 \leq X \leq 15)$, (iii) $P(X \geq 15)$.
- (b) Suppose 1 per cent of the items made by a machine are defective, find the probability that 3 or more items are defective in a sample of 100 items.

15. (a) Calculate the first four moments of the following distribution about the mean and hence find β_1 and β_2 .

x	:	0	1	2	3	4	5	6	7	8
f	:	1	8	28	56	70	56	28	8	1

(b) Derive M.G.F. and C.G.F. of chi-square distribution.

16. The no. of automobile accidents for week in a certain community are as follows:

12, 8, 20, 2, 14, 10, 15, 6, 9, 4

Are these frequencies in agreement with the belief that accidents condition were the same during this 10 week period (Table value of $\chi^2 = 16.9$).

17. (a) State and prove Taylor's theorem.

(b) Expand $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region $|z| < 1$.

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