

FACULTY OF ENGINEERING

B.E. 2/4 (ECE/M/P/CSE) II Semester Main Examination, April/May-2007

MATHEMATICS—IV

Time : Three Hours]

[Maximum Marks : 75

Note :—Answer ALL the questions from Part-A and any FIVE from Part-B.

PART—A

Choose the correct answer from the following :—

1. The value of 'a' so that $u(x, y) = ax^2 + y^2 + xy$ is harmonic :

(a) -1

(b) 1

(c) 2

(d) none

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2. The value of $\int_0^{1+i} (x^2 + iy) dz$ along the path $y = x$ is :

(a) $\frac{5}{6} + \frac{i}{6}$

(b) $\frac{5}{6} - \frac{i}{6}$

(c) $\frac{5}{6}i + 1$

(d) none

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3. Using Cauchy's integral formula, the value of $\frac{1}{2\pi i} \int_C \frac{z^2 + 5}{z - 3} dz$ where $C : |z| = 2$ is :

(a) $28\pi i$

(b) $14\pi i$

(c) 14

(d) 0

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4. The zero of $f(t) = \frac{z^3 + 1}{z^3 - 1}$ is :

(a) -1

(b) $-1 + i$

(c) $1 - i$

(d) none

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5. (i) Cauchy-Riemann equation in Polar form is _____.

(ii) The residue of $\frac{1 + e^z}{z \cos t + \sin z}$ at $z = 0$ is _____.

6. A random variable 'X' has the following probability function :

Values of X (x)	P(x)
0	0
1	k
2	2 k
3	2 k
4	3 k
5	k^2
6	$2 k^2$
7	$7 k^2 + k$

(i) Then the value of k

(ii) The value of $P(x < 6)$

(iii) $P(x \geq 6)$.

7. Indicate whether the following statements are true or false :—

(i) $f(t) = |z|^2$ is analytic function in every point of region.

(ii) If 'a' is constant and x be a random variable then $v(ax) = av(x)$.

(iii) If A and B are two events such that $P(A) = \frac{1}{3}$; $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$ then

$$P\left(\frac{B}{A}\right) \text{ is } \frac{1}{2}.$$

8. The first four moments of a distribution, about the value 5 of the variable are 2, 20, 40 and 50. Find the moments about mean.

9. Find the mean and variance of Poisson distribution.

10. Is the following statement true? Give reasons. $40x - 18y = 5$ and $8x - 10y + 6 = 0$ are respectively the regression equation of y on x is x on y.

PART—B

- 11. (a) Derive Cauchy-Riemann equations in Cartesian form.
- (b) Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate.
- 12. State Cauchy's residue theorem and hence evaluate using theorem :

(i) $\int_0^{2\pi} \frac{d\theta}{5 + 4 \sin \theta}$ (ii) $\int_0^{\infty} \frac{dx}{1 + x^4}$.

- 13. (a) Prove that :

(i) $\sigma^2 = \mu_2^1 - (\mu_1^1)^2$ (ii) $\mu_r = \mu_r^1 - r\mu_1 \mu_{r-1}^1 + \dots$

- (b) A continuous random variable 'X' defined by :

$$f(x) = \begin{cases} \frac{(3+x)^2}{16} & ; \text{if } -3 \leq x < -1 \\ \frac{6-2x^2}{16} & ; \text{if } -1 \leq x < 1 \\ \frac{(3-x)^2}{16} & ; \text{if } 1 \leq x \leq 3. \end{cases}$$

Verify that : (i) The area under the curve is unity (ii) Also show that the mean is zero.

- 14. (a) In a normal distribution exactly 7% of the items are under 35 and 89% are under 63. Find the mean and standard deviation of the distribution.
- (b) Write the properties of Normal distribution.
- 15. A survey of 320 families with 5 children each revealed the following information :

No. of boys	No. of girls	No. of families
5	0	14
4	1	56
3	2	110
2	3	88
1	4	40
0	5	12

Is this result consistent with the hypothesis that male and female births are equally probable ?

(Table value of χ^2 for 5 d.f. is 11.07)

16. (a) State and prove Cauchy's integral formula.

(b) If $f(x) = \begin{cases} \frac{x+1}{2} & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

where 'x' have Probability density function. Find the mean and standard deviation of x.

17. (a) State and prove Baye's theorem.

(b) Expand $\frac{1}{z^2 - 3z + 2}$ is Laurent's series valid with in the region $1 < |z| < 2$.