Roll No. ....

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BT-3/D08
MATHEMATICS—III
Paper : Math-201E

Time : Three Hours]   [Maximum Marks : 100

Note : Attempt *five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

UNIT–I

1. (a) Obtain the Fourier series for the function
   $f(x) = \pi x$, $0 \le x \le 1$ and $f(x) = \pi(2 - x)$, $1 \le x \le 2$.

   (b) Find the half-range cosine series for $f(x) = x^2$ in the range $0 \le x \le \pi$.

2. (a) Find the Fourier transform of $f(x) = 1 - x^2$ if $|x| < 1$ and $f(x) = 0$ if $|x| > 1$, and hence evaluate
   $$\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2}\, dx.$$

   (b) Solve the heat equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$, $x > 0$, $t > 0$ when

   (i) $u = 0$ for $x = 0$, $t > 0$.
   (ii) $u(x, t)$ is bounded.
   (iii) $u(x, 0) = \begin{cases} 1, & 0 \le x < 1 \\ 0, & x \ge 1. \end{cases}$

UNIT–II

3. (a) If $\cos(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that
   (i) $\sin^2\theta = \pm\sin\alpha$
   (ii) $\cos 2\theta + \cosh 2\phi = 2$.

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(b) If $u = \log \tan \left(\dfrac{\pi}{4} + \dfrac{\theta}{2} \right)$, then prove that

    (i) $\tanh u/2 = \tan \theta/2$.

    (ii) $\cosh u = \sec \theta$.

4. (a) Determine the analytic function whose real part is $u = e^x (x \cos y - y \sin y)$.

  (b) Show that the transformation $w = \dfrac{i(1 - z)}{1 + z}$ maps the region $|z| \leq 1$ into the upper half of w-plane including the real axis.

UNIT–III

5. (a) A survey was conducted to find the supplier of the consumer durables for the market. It was found that the three major companies A, B and C have market share of 35%, 25% and 40% respectively out of which 2%, 1% and 3% are not upto the satisfaction. A consumer buys a product and is dissatisfied with it. What is the probability that it might be from the company C ?

  (b) Obtain the m.g.f. of the random variable X having p.d.f.
$f(x) = x$ for $0 \leq x < 1$, $f(x) = 2 - x$ for $1 \leq x < 2$ and $f(x) = 0$ elsewhere.
Determine μ'_1 and μ_2.

6. (a) Show that the mean deviation from the mean of a normal probability distribution is approximately $\dfrac{4}{5}$ times the standard deviation.

  (b) If the mean of a binomial distribution is 3 and the variance is $\dfrac{3}{2}$, find the probability of obtaining at least 4 successes.

UNIT–IV

7. (a) A firm manufactures two items A and B. It purchases castings which are then machined, bored and polished. Castings for items A and B cost Rs. 3 and Rs. 4 each and are sold at Rs. 6 and Rs. 7 respectively. Running costs of these machines are Rs. 20, Rs. 14 and Rs. 17.50 per hour respectively. Formulate the problem so that the product mix maximizes the profit.

The capacities of the machines are

	Item A	Item B
Machining	25 per hr.	40 per hr.
Boring	28 per hr.	35 per hr.
Polishing	35 per hr.	25 per hr.

(b) Using Graphical method, solve the following problem :

Minimize $\quad Z = 20x + 10y$

subject to $\quad x + 2y \leq 40 \quad\quad 3x + y \geq 30$

$\quad\quad\quad 4x + 3y \geq 60 \quad\quad x, y \geq 0.$

8. (a) Use Simplex method to solve the following problem :

Maximize $\quad Z = x_1 + x_2 + 3x_3$

sub. to $\quad 3x_1 + 2x_2 + x_3 \leq 3$

$\quad\quad\quad 2x_1 + x_2 + 2x_3 \leq 2$

$\quad\quad\quad\quad x_1, x_2, x_3 \geq 0.$

(b) Solve the following LPP by Dual simplex method :

Minimize $\quad Z = 2x_1 + 2x_2 + 4x_3$

sub. to $\quad 2x_1 + 3x_2 + 5x_3 \geq 2$

$\quad\quad\quad 3x_1 + x_2 + 7x_3 \leq 3$

$\quad\quad\quad x_1 + 4x_2 + 6x_3 \leq 5$

$\quad\quad\quad\quad x_1, x_2, x_3 \geq 0.$