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BT–4/M09 9399

Mathematics—III

Paper : Math-201E

Time : Three Hours] [Maximum Marks : 100

Note :— Attempt any **FIVE** questions, selecting at least **ONE** from each section.

SECTION—I

1. (a) Find a Fourier series for the function defined by :

$$f(x) = \begin{cases} -1 \text{ for } & -\pi < x < 0 \\ 0 \text{ for } & x = 0 \\ 1 \text{ for } & 0 < x < \pi. \end{cases}$$

Hence prove that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots \infty = \pi/4.$$ 10

(b) Write conditions for a Fourier expansion. Hence obtain the Fourier series for the function :

$$f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2 - x) & 1 \le x \le 2. \end{cases}$$ 10

2. (a) Solve the integral equation $\int_0^\infty f(x) \cos \alpha x \, dx = e^{-\alpha}$.

(b) Using Fourier transformation, solve the differential equation :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \text{ given that :}$$

$u(0, t) = 0$

$u(u, t) = 0$

$u(x, 0) = 2x;\ 0 < x < u,\ t > 0.$ 20

SECTION—II

3. (a) If $\mu = \log \tan (\pi/4 + \theta/2)$, prove that

$$\theta = -i \log \tan \left(\pi/4 + \frac{i\mu}{2} \right).$$ 8

(b) Show that $f(z) = \begin{cases} xy^2 (x + iy)/x^2 + y^4, & z \neq 0 \\ 0 & , z = 0 \end{cases}$

is not analytic at $z = 0$, although C.R. equations are satisfied at the origin. 12

4. Discuss fully the transformation $w = c \cosh z$, where c is a real number. 20

SECTION—III

5. (a) Given that $P(A) = 1/2$, $P(B) = 1/3$, $P(A \cap B) = 1/4$. Find $P(A \mid B)$, $P(A \cup B)$, $P(A' \mid B')$. 10

(b) A die is tossed thrice. A success is "getting 1 or 6" on a toss. Find the mean and variance of the number of successes. 10

6. (a) Find the moment generating function of the exponential distribution :

$$f(x) = \frac{1}{c} e^{-x/c}, 0 \leq x \leq \infty, c > 0.$$

Hence find its mean and S.D. 10

(b) Fit a Poisson distribution to the following :

x	f
0	46
1	38
2	22
3	9
4	1

10

SECTION—IV

7. (a) Using Graphical Method, solve :

$$\text{Min. } Z = 20x_1 + 30x_2$$
$$\text{sub. to } \quad x_1 + 2x_2 \le 40$$
$$3x_1 + x_2 \ge 30$$
$$4x_1 + 3x_2 \ge 60$$
$$x_1, x_2 \ge 0.$$
10

(b) Find the solution space of the L.P.P. :

$$\text{Max. } \quad Z = x + 3y + 3z$$
$$\text{sub. to } \quad x + 2y + 3z = 4$$
$$2x + 3y + 5z = 7$$
$$x, y, z \ge 0.$$

Which of these are the (a) basic, (b) non-degenerate basic feasible, (c) optimal basic feasible ? 10

8. (a) Using Simplex Method, solve

$$\text{Max. } Z = x + 3y$$
$$\text{sub. to } \quad x_1 + 2y \le 10$$
$$0 \le x \le 5$$
$$0 \le y \le 4$$
10

(b) Using Dual Simplex Method, solve :

$$\text{Min. } Z = 2x + 2y + 4z$$
$$\text{sub. to } \; 2x + 3y + 5z \ge 2$$
$$3x + y + 7z \le 3$$
$$x + 4y + 6z \le 5$$
$$x, y, z \ge 0.$$
10