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BT–4/J08 50 8752

Mathematics—III

Paper : Math-201E

Option—II

Time : Three Hours] [Maximum Marks : 100

Note :— Attempt any **FIVE** questions, selecting at least **ONE** from each section.

SECTION—A

1\. (a) Find the Fourier series to represent the function f(x) given by

$$F(x) = x \quad , \quad 0 \le x \le \pi$$
$$= 2\pi - x \, , \quad \pi \le x \le 2\pi.$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots \infty = \frac{\pi^2}{8}$. 10

(b) Obtain a Half-Range cosine series for :

$$F(x) = kx \quad , \quad 0 \le x \le \frac{\ell}{2}$$
$$= k(\ell - x) \quad , \quad \frac{\ell}{2} \le x \le \ell.$$ 10

2\. (a) Find the Fourier Transform of

$$F(x) = \begin{cases} 1 - x^2 & ; \quad |x| < 1 \\ 0 & ; \quad |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos x \, dx$. 10

(b) If the initial temperature of an infinite bar is given by

$$\theta(x) = \begin{cases} \theta_0 & \text{for } |x| < a \\ 0 & |x| > a \end{cases}$$

determine the temperature at any point x and at any instant t. 10

SECTION—B

3. (a) If tan (A + iB) = x + iy, prove that

(i) $x^2 + y^2 + 2x \cot 2A = 1$.

(ii) $x^2 + y^2 - 2y \coth 2B + 1 = 0$. 5+5

(b) Determine the Analytic function whose Real part is $e^x (x \cos y - y \sin y)$. 10

4. (a) If f(z) is a Holomorphic function of z, show that

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2.$$ 10

(b) Show that the transformation

$W = z + \dfrac{a^2 - b^2}{4z}$ transforms the circle of radius $\dfrac{1}{2}(a + b)$, centre at the origin, in the z-plane into ellipse of semi-axes a, b in the w-plane. 10

SECTION—C

5. (a) There are three bags : First containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls.

Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the second bag. 10

(b) X is a continuous random variable with probability density function given by

$$f(x)=\begin{cases} kx & 0 \le x < 2 \\ 2x & 2 \le x < 4 \\ -kx+6k & 4 \le x < 6 \end{cases}$$

Find the mean value of X and value of k. 10

6. (a) Fit the Poisson Distribution to the following :

x	f
0	46
1	38
2	22
3	9
4	1

10

(b) Show that the Standard Deviation for a Normal Distribution is approximately 25% more than the mean deviation. 10

SECTION—D

7. (a) Graphically solve the LPP :

Max. $Z = 6x_1 + 4x_2$

subject to $2x_1 + 3x_2 \le 30$

$3x_1 + 2x_2 \ge 24;$

$x_1 + x_2 \ge 3,$

$x_1, x_2 \ge 0.$ 10

(b) Solve the following LPP by Simplex Method :

Max. $Z = 5x_1 + 3x_2$

subject to $x_1 + x_2 \le 2$;

$5x_1 + 2x_2 \le 10$

$3x_1 + 8x_2 \le 12$;

$x_1, x_2 \ge 0$. 10

8. (a) Using Dual Simplex Method :

Minimize $Z = x_1 + 2x_2 + 3x_3$.

subject to $2x_1 - x_2 + x_3 \ge 4$;

$x_1 + x_2 + 2x_3 \le 8$;

$x_2 - x_3 \ge 2$;

$x_1, x_2, x_3 \ge 0$. 10

(b) Define :—

(i) Basic Feasible Solutions.

(ii) Optimum Basic Feasible Solution.

(iii) Slack/Surplus Variable.

(iv) Artificial Variable.

(v) Degeneracy in LPP. 2×5