Roll No. [redacted] Total Pages : 3

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BTI/D09

MATHEMATICS-I

(2004-05 Onwards)

Paper : MATH-101E

Opt. (ii)

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) Find the radius and centre of curvature for the curve $x^3 + y^3 = 3xy$ at the point (3/2, 3/2) on it.

(b) Using Maclaurin's series, prove that :

$$e^{x\sin x} = 1 + x^2 + \frac{x^4}{3} + \frac{x^6}{120} + \cdots.$$

2. (a) Trace the curve :

$y = (x^2 + 1)/(x^2 - 1)$.

(b) Find the asymptotes of the curve :

$r\sin\theta = 2\cos 2\theta$.

UNIT–II

3. (a) State Euler's Theorem for a homogeneous function of two variables. Given :

$$Z = x^n f_1\left(\frac{y}{x}\right) + y^{-n} f_2\left(\frac{x}{y}\right), \text{ prove that}$$

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = n^2 z.$$

(b) If $u = x + y + z$, $uv = y + z$, $uvw = z$, show that $\partial(x, y, z)/\partial(u, v, w) = u^2 v$.

4. (a) The range R of a projectile which starts with a velocity v at an elevation α is given by $R = \frac{v^2 \sin 2\alpha}{g}$. Find the percentage error in R due to an error of 1% in v and an error of 0.5% in α.

(b) Using the method of differentiation under the sign of integration, evaluate

$$\int_0^{\infty} e^{-x^2} \cos \alpha x \, dx.$$

UNIT–III

5. (a) Evaluate :

$\iint (x + y)^2 \, dx \, dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

(b) Using Double integration, prove that the volume generated by the revolution of the cardioid $r = a(1 + \cos\theta)$ about its axis is $8\pi a^3/3$.

6. (a) Evaluate :

$$\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dz \, dy \, dx.$$

(b) Prove that :

$$\beta(m, n) = \frac{\gamma(m)\gamma(n)}{\gamma(m+n)}.$$

UNIT-IV

7\. (a) If $\vec{A}$ is a constant vector and $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that :

(i) Curl $(\vec{A} \times \vec{R}) = 2\vec{A}$.

(ii) Curl $[(\vec{A} \cdot \vec{R})\vec{R}] = \vec{A} \times \vec{R}$.

(b) Define gradient of a scalar point function and give its geometrical interpretation.

8\. (a) Using Stoke's Theorem evaluate

$$\int_C [(x+y)dx + (2x-z)dy + (y+z)dz],$$

where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0 6).

(b) Use Divergence Theorem to evaluate :

$$\iint (x\,dy\,dz + y\,dz\,dx + z\,dx\,dy)$$

over the surface of a sphere radius a.
