Roll No.

Total Pages: 3

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BTI/D09

MATHEMATICS-I

(2004-05 Onwards)

Paper: MATH-101E

Opt. (ii)

Time: Three Hours]

[Maximum Marks: 100

Note: Attempt five questions in all, selecting at least one question from each unit. All questions carry equal marks.

UNIT-I

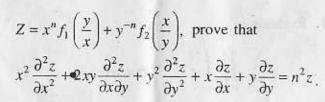
- 1. (a) Find the radius and centre of curvature for the curve $x^3 + y^3 = 3xy$ at the point (3/2, 3/2) on it.
 - (b) Using Maclaurin's series, prove that :

$$e^{x\sin x} = 1 + x^2 + \frac{x^4}{3} + \frac{x^6}{120} + \cdots$$

- 2. (a) Trace the curve : $y = (x^2 + 1)/(x^2 1)$.
 - (b) Find the asymptotes of the curve : $r \sin \theta = 2 \cos 2\theta$.

UNIT-II

3. (a) State Euler's Theorem for a homogeneous function of two variables. Given:



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(b) If
$$u = x + y + z$$
, $uv = y + z$, $uvw = z$, show that $\partial(x, y, z)/\partial(u, v, w) = u^2v$.

- 4. (a) The range R of a projectile which starts with a velocity v at an elevation α is given by $R = \frac{v^2 \sin 2\alpha}{g}$. Find the percentage error in R due to an error of 1% in v and an error of 0.5% in α .
 - (b) Using the method of differentiation under the sign of integration, evaluate

$$\int_{0}^{\infty} e^{-x^{2}} \cos \alpha x \, dx.$$

UNIT-III

- 5. (a) Evaluate: $\iint (x+y)^2 dx dy \text{ over the area bounded by the ellipse}$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$
 - (b) Using Double integration, prove that the volume generated by the revolution of the cardioid $r = a(1 + \cos\theta)$ about its axis is $8\pi a^3/3$.
- 6. (a) Evaluate:

$$\iint\limits_{0}^{a} \int\limits_{0}^{x} \int\limits_{0}^{x+y} e^{x+y+z} dz \, dy \, dx \, \cdot$$

(b) Prove that :

$$\beta(m,n) = \frac{\gamma(m)\gamma(n)}{\gamma(m+n)}$$

UNIT-IV

- 7. (a) If \vec{A} is a constant vector and $\vec{R} = x \hat{i} + y \hat{j} + z \hat{k}$, prove that:
 - (i) Curl $(\vec{A} \times \vec{R}) = 2\vec{A}$,
 - (ii) Curl $[(\vec{A} \cdot \vec{R})\vec{R}] = \vec{A} \times \vec{R}$.
 - (b) Define gradient of a scalar point function and give its geometrical interpretation.
- 8. (a) Using Stoke's Theorem evaluate

$$\iint_C [(x+y)dx + (2x-z)dy + (y+z)dz],$$

where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0 6).

(b) Use Divergence Theorem to evaluate :

$$\int \int (x \, dy \, dz + y \, dz \, dx + z \, dx \, dy)$$

over the surface of a sphere radius a.