Model Question Paper
Quantitative Techniques – I (MSF1B1)

- Answer all 78 questions.
- Marks are indicated against each question.

Total Marks : 100

1. A box contains 3 red balls and 7 green balls. If two balls are taken one after the other without replacement, the probability that both are red is

(a) $\frac{1}{2}$

(b) $\frac{3}{10}$

(c) $\frac{1}{15}$

(d) $\frac{1}{40}$

(e) $\frac{6}{19}$.

(1 mark)

2. Which of the following is/are **true** regarding probability?

I. The probability of the entire sample space S is 1.
II. The probability of an event contained in the sample space should be greater than or equal to 0 and less than or equal to 1.
III. If A and B are dependent events, then the probability of (A and B) is equal to the product of probabilities of A and B.

(a) Only (I) above
(b) Only (II) above
(c) Only (III) above
(d) Both (I) and (II) above
(e) All (I), (II) and (III) above. **(1 mark)**

3. The simplified form of the expression $\sqrt[6]{a^{4b}\,x^{6}} \times \left(a^{\frac{2}{3}}\,x^{8}\right)^{-b}$ is

(a) x^{1-b}

(b) x^{1-6b}

(c) x^{1-8b}

(d) x^{6-8b}

(e) x^{2-8b}.

(1 mark)

4. If $\log\frac{X+Y}{7} = \frac{1}{2}(\log X + \log Y)$ then $\frac{X}{Y}+\frac{Y}{X}$ is equal to

(a) 16
(b) 27
(c) 38
(d) 47
(e) 50. **(2marks)**

5. The harmonic mean of two numbers is 4. The arithmetic mean and geometric mean of the same numbers are A and G respectively. The relationship between A and G is $2A + G^2 = 27$. One of the two numbers is

(a) 3
(b) 4
(c) 5
(d) 7
(e) 8. **(1 mark)**

6. The product of three numbers in a Geometric Progression (G.P) is 216 and the sum of their products taken in pairs is 156. The highest of the three numbers is

(a) 8
(b) 10
(c) 18
(d) 38
(e) 48. **(2marks)**

7. The number of combinations that can be made by taking 4 letters of the word COMBINATION is

(a) 80
(b) 136
(c) 230
(d) 330
(e) 430. **(2marks)**

8. The mean and standard deviations of the following observations 3, 7, 12, 15, 18, 21 are 12.6667, 6.18 respectively, if each observation is increased by 2.5, then the mean and standard deviation are

(a) 12.6667, 6.18
(b) 15.1667, 6.18
(c) 23.1667, 8.68
(d) 25.1667, 6.18
(e) 32.1667, 8.68. **(1 mark)**

9. An organization has 30 members 5 of them are women in how many ways a committee of 4 members is selected so as to include at least 1 woman?

(a) 12,547
(b) 14,755
(c) 13,526
(d) 15,874
(e) 16,875. **(1 mark)**

10. The intercept for the line joining points (1, 2) and (3, 4) is

(a) $\frac{4}{3}$
(b) $\frac{5}{3}$
(c) 1
(d) $\frac{8}{3}$
(e) $\frac{10}{3}$. **(1 mark)**

11. Consider the following equations:

$$\frac{1}{2}x+\frac{1}{2}y-\frac{1}{6}z=5$$
$$\frac{1}{3}x+\frac{1}{4}y+\frac{1}{2}z=10$$
$$\frac{1}{6}x+\frac{1}{4}y+\frac{1}{3}z=7.$$

The value of z in the above simultaneous equations is

(a) 10
(b) 11
(c) 12
(d) 13
(e) 14. **(1 mark)**

12. If ${}^{n}C_{6} : {}^{n-3}C_{3} = 33:4$. Then the value of n is

(a) 10
(b) 11
(c) 12
(d) 14
(e) 16. **(2marks)**

13. The roots of the quadratic equation $x^2 + 7x + 12 = 0$ are

(a) – 1 and – 2
(b) – 2 and – 3
(c) – 1 and – 4
(d) – 1 and – 3
(e) – 3 and – 4. **(1 mark)**

14. The factors of $16Q^2 - 81$ is

(a) (2Q + 8)(2Q – 8)
(b) (2Q + 9)(2Q – 9)
(c) (2Q + 9)(4Q – 8)
(d) (4Q + 8)(4Q – 8)
(e) (4Q + 9)(4Q – 9). **(1 mark)**

15. What is the value of ${}^{8}P_{6}$?

(a) 20,160
(b) 30,420
(c) 41,256
(d) 45,217
(e) 48,217. **(1 mark)**

16. If $x^y = \log_e x$, then $\frac{dy}{dx}$ at x = e is

(a) $\frac{1+y}{e}$

(b) $\frac{1-y}{e}$

(c) $\frac{2+y}{e}$

(d) 1
(e) 0. **(1 mark)**

17. The limit of the function $\frac{2x^2 - x - 3}{x^2 - 2x - 3}$ at x = –1 is

(a) $\frac{5}{4}$

(b) $\frac{5}{2}$

(c)

$\frac{1}{4}$

(d) 0

(e) 1. **(1 mark)**

18. The total costs of ordering and carrying costs are given as TC = $\frac{10^2}{Q} + \frac{2.25Q}{2}$ where Q is the quantity per order. The order quantity for which the total cost will be minimum is

(a) 6.4251
(b) 9.4281
(c) 8.2569
(d) 7.2564
(e) 5.2316. **(2marks)**

19. Which of the following is/are **true** regarding interpolation and extrapolation?

I. Interpolation and extrapolation provide us with estimates of independent variable.
II. There must exist a functional relationship between interpolation and extrapolation for them to work.
III. Interpolation is a statistical technique, which through a study of time series of known figures of population, allows us to make insertions.
IV. Interpolation allows us to forecast or anticipate a value for some future data.
V. In interpolation or extrapolation problem, time can be taken as one of the dependent variables.

(a) Only (I) above
(b) Only (III) above
(c) Both (I) and (III) above
(d) Both (II) and (III) above
(e) All (I), (II), (III), (IV) and (V) above. **(1 mark)**

20. Which of the following is **false** with regard to histograms?

(a) The class intervals are represented by the base of the rectangles
(b) The frequencies are represented by the heights of the rectangles
(c) If the class intervals are of equal width then the bases of the rectangles will be equal
(d) The tallest rectangle represents the class interval with the lowest frequency
(e) The tallest rectangle represents the class interval with highest frequency. **(1 mark)**

21. A project requires an initial outlay of Rs.35 lakhs and has the following cost flow projections:

Year	Cash flows (in Rs. lakhs)
1	15
2	10
3	10
4	20

Internal Rate of Return (IRR) is

(a) 12.31%
(b) 14.31%
(c) 15.81%
(d) 18.85%
(e) 19.82%. **(2marks)**

22. A graph of cumulative frequency distribution is called as

(a) Histogram
(b) Frequency polygon
(c) Ogive
(d) Skewed curve
(e) Frequency curve. **(1 mark)**

23. A relative frequency polygon indicates the _____ of the distribution.

(a) Skewness
(b) Mean
(c) Median
(d) Mode
(e) Variance. **(1 mark)**

24. A positively skewed distribution curve tails off towards the higher end, and for such a curve

(a) Mean = Median = Mode
(b) Mean < Median = Mode
(c) Mean = Median > Mode
(d) Mean < Median > Mode
(e) Mean > Median > Mode. **(1 mark)**

25. The sum of deviations of the items from the arithmetic mean is always

(a) One
(b) Two
(c) Three
(d) Four
(e) Zero. **(1 mark)**

26. A problem in statistics is given to two students A and B. The probability of A solving the problem is 2/5 and the probability of B solving the problem is 5/11. If A and B attempt, what is the probability of the problem being solved?

(a) 0.253
(b) 0.453
(c) 0.673
(d) 0.783
(e) 0.853. **(2marks)**

27. Bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. What is the probability that it was drawn from bag B?

(a) $\frac{15}{52}$

(b) $\frac{25}{52}$

(c) $\frac{35}{52}$

(d) $\frac{45}{52}$

(e) $\frac{50}{52}$. **(2marks)**

28. The probability that a boy will get a scholarship is 0.9 and that a girl will get is 0.8. What is the probability that at least one of them will get the scholarship?

(a) 0.45
(b) 0.65
(c) 0.75
(d) 0.85
(e) 0.98. **(2marks)**

29. A train runs 25 miles at a speed of 30 mph, another 50 miles at a speed of 40 mph, then due to repair of the tracks it travels for 6 minutes at a speed of 10 mph and finally covers the remaining distance of 24 miles at a speed of 24 mph. What is the average speed in miles per hour?

(a) 31.41
(b) 41.21
(c) 51.22
(d) 62.22
(e) 68.66. **(2marks)**

30. The value of the median of the following data would be

Income (Rs.)	1,000	1,500	800	2,000	2,500	1,800
No. of persons	24	26	16	20	6	30

(a) Rs. 150
(b) Rs. 800
(c) Rs.1,000
(d) Rs.1,500
(e) Rs.1,900. **(1 mark)**

31. In a certain city two news papers A and B are published. It is known that 25% of the city population reads A and 20% reads B, while 8% reads both A and B. It is also known that 30% of those who read A but not B look into advertisement and 40% of those who read B but not A look advertisements while 50% of those who read both A and B look into advertisement. What is the percentage of the population who look into advertisement?

(a) 11.2%
(b) 12.3%
(c) 12.8%
(d) 13.9%
(e) 24.5%. **(2marks)**

32. The function $y = x^3 - 8x^2 + 21x - 18$ has a minimum value at

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5. **(2marks)**

33. Which of the following is **true** when $f'(x) > 0$ and $f''(x) > 0$?

(a) f(x) is increasing at an increasing rate
(b) f(x) is decreasing at an increasing rate
(c) f(x) is increasing at a decreasing rate
(d) f(x) is decreasing at a decreasing rate
(e) f(x) is decreasing at a constant rate. **(1 mark)**

34. Which of the following is **most** affected by the presence of extreme values in the data set?

(a) Mean
(b) Median
(c) Mode
(d) Quartiles
(e) Deciles. **(1 mark)**

35.

If $U = \log (x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2)$, then $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} + z\frac{\partial U}{\partial z} =$

(a) 4
(b) 2U
(c) 3U
(d) 4U
(e) 5U. **(1 mark)**

36. Consider the frequency distribution

Wages in (Rs.)	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45
No. of workers	60	140	110	150	120	100	90

The modal wage of the workers is

(a) 15.8671
(b) 20.8671
(c) 27.8571
(d) 35.8571
(e) 45.8671. **(1 mark)**

37. The harmonic mean from the following data is

25	45	75	5	10	20	50	60

(a) 14.5231
(b) 15.6287
(c) 16.2987
(d) 17.3085
(e) 19.6234. **(1 mark)**

38. The variance of a group values is 12.25. If every observation is increased by 3 the variance of the resulting values

(a) Decreases by 3
(b) Increases by 9
(c) Decreases by 9
(d) Remains 12.25
(e) Increases by 3. **(1 mark)**

39. The derivative of a function, y = f (x), at a point x = a is equal to

(a) The limit of the function at x = a
(b) The value of the function at x = a
(c) $\lim_{x \to a} \frac{f(x)}{a}$
(d) $\lim_{x \to a} \frac{f(x)}{f(a)}$
(e) $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$. **(1 mark)**

40. If for a sample of 10 items, $\sum x^2 = 385$ and $\bar{x} = 5.5$, then the standard deviation of the sample is

(a) 1.0377
(b) 2.0377
(c) 3.0277
(d) 4.0377
(e) 5.0377. **(1 mark)**

41. Consider the following distribution:

Percentage of dividend declared	Number of companies
10 – 20	6
20 – 30	15
30 – 40	10
40 – 50	14
50 – 60	11

What is the third quartile for the above data?

(a) 23.5645
(b) 28.5236
(c) 47.8571
(d) 33.5845
(e) 43.5954. **(2marks)**

42. Data on the readership of a certain magazine show that the proportion of male readers under 35 is 0.40 and over 35 is 0.20. If the proportion of readers under 35 is 0.70, what is the probability that a randomly selected male reader is under 35 years of age?

(a) 0.67
(b) 0.77
(c) 0.87
(d) 0.97

(e) 0.99. **(1 mark)**

43. Consider the following 2 sets of observations.

Set 1 : 3, 10, 12
Set 2 : 6, 4, 1, 9

The combined standard deviation of the data is

(a) 1.736
(b) 2.736
(c) 3.736
(d) 4.736
(e) 5.736. **(2marks)**

44. Which of the following statements is **false**?

(a) If the primal formulation of a linear programming problem is a minimizing problem then the dual formulation will be a maximizing problem
(b) If the primal formulation of a linear programming problem is a maximizing problem then the dual formulation will be a minimizing problem
(c) The dual formulation is a inverse of the primal formulation in every respect
(d) The dual formulation cannot be solved for optimal solution
(e) The dual formulation can be derived from the same data from which the primal is formulated. **(1 mark)**

45. The feasible region for a profit maximizing linear programming is bounded by the vertices (1, 0), (6, 0), (4, 9) and (0, 15). The profit function is Z = 2x + 3y. The maximum value of Z is

(a) 2
(b) 12
(c) 35
(d) 45
(e) 65. **(1 mark)**

46. If the objective function in a graphical linear programming problem is parallel to an edge of the feasible region then

(a) The optimal solution lies outside the feasible region
(b) There cannot be any optimal solution
(c) There will be a unique optimal solution
(d) The optimal solution is always one of the corner points
(e) There will be multiple number of optimal solutions. **(1 mark)**

47. Consider the following Linear Programming Problem(LPP):

Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to constraints

$x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_3 \leq 420$

And $x_1, x_2, x_3 \geq 0$.

The optimum value of z by using simplex method is

(a) 117
(b) 277
(c) 310
(d) 950
(e) 1,050. **(3marks)**

48. A company manufactures two types of printed circuits. The requirements of transistors, resistors and capacitors for each type of printed circuits along with the other data are given below:

Particulars	Circuit		Stock available (in units)
	A	B	

Transistor	15	10	180
Resistor	10	20	200
Capacitor	15	30	210
Profit (Rs.)	5	8	

What is the optimum value of the above problem by graphical method?

(a) 60
(b) 76
(c) 80
(d) 67
(e) 100. **(2marks)**

49. The reciprocals of the terms in a harmonic progression are

(a) In geometric progression
(b) In harmonic progression
(c) In arithmetic progression
(d) Always in decreasing order
(e) Always in increasing order. **(1 mark)**

50. The inverse of the function $y = f(x) = 7x - 5$ is

(a) $\frac{y+7}{4}$

(b) $\frac{y-7}{4}$

(c) $\frac{y+7}{7}$

(d) $\frac{y+5}{7}$

(e) $\frac{y+7}{5}$. **(1 mark)**

51. Consider the following LPP :

Minimize $Z = 2x_1 + 3x_2 + 4x_3$

Subject to constraints:

$x_1 + 2x_2 + 3x_3 \geq 5$

$3x_1 + 4x_2 + 5x_3 \geq 6$

$6x_1 + 7x_2 + 8x_3 \geq 8$

And $x_1, x_2, x_3 \geq 0$

The dual of the above LPP is

(a) Maximize $Z^* = 5y_1 + 6y_2 + 8y_3$
Subject to constraints:
$y_1 + 3y_2 + 6y_3 \leq 2$
$2y_1 + 4y_2 + 7y_3 \leq 3$
$3y_1 + 5y_2 + 8y_3 \leq 4$
And $y_1, y_2, y_3 \geq 0$

(b) Maximize $Z^* = 5y_1 + 6y_2 + 8y_3$
Subject to constraints:
$y_1 + 3y_2 + 6y_3 \leq 2$
$2y_1 + 4y_2 + 6y_3 \leq 3$
$3y_1 + 5y_2 + 8y_3 \leq 4$
And $y_1, y_2, y_3 \geq 0$

(c) Maximize $Z^* = 5y_1 + 8y_2 + 6y_3$
Subject to constraints:
$y_1 + 3y_2 + 6y_3 \leq 2$
$2y_1 + 4y_2 + 7y_3 \leq 3$
$3y_1 + 5y_2 + 8y_3 \leq 4$
And $y_1, y_2, y_3 \geq 0$

(d) Maximize $Z^* = y_1 + 6y_2 + 3y_3$
Subject to constraints:
$y_1 + 3y_2 + 6y_3 \leq 2$
$2y_1 + 4y_2 + 7y_3 \leq 3$
$3y_1 + 5y_2 + 8y_3 \leq 4$
And $y_1, y_2, y_3 \geq 0$

(e) Maximize $Z^* = 5y_1 + 6y_2 + 8y_3$
Subject to constraints:
$y_1 + 3y_2 + 6y_3 \leq 2$
$2y_1 + 4y_2 + 7y_3 \leq 3$
$3y_1 + 5y_2 + 8y_3 \geq 4$
And $y_1, y_2, y_3 \geq 0$.

(2marks)

52. $x = \frac{\sqrt[9]{81} \times \sqrt[7]{49}}{\sqrt[3]{25}}$, $y = \frac{\sqrt[9]{625} \times \sqrt[3]{3}}{\sqrt[7]{7^3}}$ and $z = \frac{\sqrt[9]{9}}{\sqrt[9]{5^7} \times \sqrt[7]{7^6}}$ then the value of xyz is

(a) $\frac{5}{21}$

(b) $\frac{7}{15}$

(c) $\frac{15}{7}$

(d) $\frac{21}{5}$

(e) $\frac{3}{35}$.

(2marks)

53. Three terms are in geometric progression. The second term and the third term are 16 and 64 respectively, then the first term is

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5.

(1 mark)

54. If mean = 29.51 and median = 39.69, then the value of empirical mode is

(a) 30.05
(b) 40.05
(c) 50.05
(d) 60.05
(e) 70.05.

(1 mark)

55. What is the value of $\log_3 81$?

(a) 4
(b) 5

(c) 6
(d) 8
(e) 9. **(1 mark)**

56. If $f(x,y,z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$ then $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} =$

(a) 0
(b) 1
(c) $f(x,y,z)$
(d) $2f(x,y,z)$
(e) $3f(x,y,z)$. **(2marks)**

57. If the ratio of the sum of m terms and n terms of an Arithmetic Progression (A.P) is $m^2 : n^2$, then the ratio of its m^{th} and n^{th} terms will be

(a) $(2m + 1) : (2n + 1)$
(b) $(2m - 1) : (2n + 1)$
(c) $(2m - 1) : (2n - 1)$
(d) $(2m + 1) : (2n - 1)$
(e) $(m + 1) : (n + 1)$. **(1 mark)**

58. According to the Bienayme-Chebyshev rule, at least _____ percent of the observations in a sample are contained within two standard deviations around the mean, regardless of the shape of the distribution.

(a) 88.89
(b) 93.75
(c) 68.26
(d) 75.00
(e) 50.00. **(1 mark)**

59. The data below represents the amount of carbohydrates in grams contained in breakfast cereal:

11, 15, 23, 29, 19, 22, 21, 20, 15, 25, 17.

The median carbohydrate amount in the cereal is

(a) 10 grams
(b) 20 grams
(c) 30 grams
(d) 40 grams
(e) 50 grams. **(1 mark)**

60. The coefficient of variation of a data set is 20 and the variance of the data set is 16. What is the mean of the data set?

(a) 20
(b) 30
(c) 40
(d) 50
(e) 60. **(1 mark)**

61. A data set contains the following observations:

10, 12, 8, 14, 15, 7, 17, 20

What is the range of the data set?

(a) 10
(b) 11
(c) 12
(d) 13
(e) 15. **(1 mark)**

62. The following distribution shows the ages of 100 persons in a group:

Age group (in years)	Number of persons
20 – 25	3
25 – 30	16
30 – 35	22
35 – 40	18
40 – 45	14
45 – 50	10
50 – 55	7
55 – 60	6
60 – 65	4

What is the average age of the persons in the group?

(a) 25.3
(b) 26.3
(c) 35.3
(d) 39.3
(e) 46.3. **(1 mark)**

63. The mean annual salary paid to 100 employees of a company was Rs.5,000. The mean annual salaries paid to male and female employees were Rs.6,000 and Rs.4,000 respectively. The percentage of males employed by the company is

(a) 40
(b) 50
(c) 60
(d) 70
(e) 80. **(2marks)**

64. The Arithmetic Mean (A.M) between two numbers exceeds their Geometric Mean (G.M) by 2 and the G.M exceeds their Harmonic Mean (H.M) by 8/5. The highest of the two numbers is

(a) 12
(b) 16
(c) 20
(d) 32
(e) 54. **(1 mark)**

65. A function, y = f (x), is said to have a relative maxima or minima at x = a, if

(a) $f'(x) = 0$ at $x = a$
(b) $f'(x) > 0$ at $x = a$
(c) $f'(x) < 0$ at $x = a$
(d) $f(x) \geq 0$ at $x = a$
(e) $f(x) < 0$ at $x = a$. **(1 mark)**

66. Which of the following is/are **true** with regard to High-level language?

I. It is a English like language with a predefined format.
II. In High-level language the instructions are not more powerful compared to machine language.
III. It is a machine independent and can be used on various computer systems.

(a) Only (I) above
(b) Only (II) above
(c) Only (III) above
(d) Both (I) and (II) above
(e) Both (I) and (III) above. **(1 mark)**

67. If the slope and intercept of a line on Y-axis are 4 and 3 respectively, then the equation of a line can be written as

(a) $Y = 4X + 3$
(b) $Y = 3X + 4$
(c) $Y = 4X^2 + 3$

(d) $Y = 3X^2 + 4$
(e) $Y = 4X^2 + 7$. **(1 mark)**

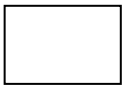
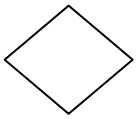
68. The limit of a function, y = f (x), at a point x = a, is the

(a) Minimum value of the function for all values of x greater than a
(b) Minimum value of the function for all values of x less than a
(c) Maximum value of the function for all values of x greater than a
(d) Maximum value of the function for all values of x less than a
(e) Value which the function approaches if x approaches a. **(1 mark)**

69. For a real number, **a**, the interval [a, + ∞) indicates

(a) Set of all real numbers less than a
(b) Set of all real numbers less than or equal to a
(c) Set of all even numbers greater than a
(d) Set of all odd numbers greater than a
(e) Set of all real number greater than or equal to a. **(1 mark)**

70. Which of the following symbols indicates the off-page connector in drawing flowchart?

(a) 
(b) 
(c) 
(d) 
(e) 

. **(1 mark)**

71. Which of the following statements is **false**?

(a) Java can be downloaded straight from internet
(b) Modem is a piece of equipment that connects a computer to a data transmission line
(c) WWW is a big part of network of networks
(d) Internet is a global network of computers linked by exclusive and regular phone lines and microwave and satellite signals
(e) Baud is a bundle of data. **(1 mark)**

72. Generally, the word size in mini computers are

(a) 8 bits
(b) 16 bits
(c) 24 bits
(d) 28 bits
(e) 32 bits or more. **(1 mark)**

73. In which of the following classification of computers uses the properties of electronic circuits to represent exactly two states – on and off?

I. Digital computer.
II. Analog computer.
III. Hybrid computer.

(a) Only (I) above
(b) Only (II) above
(c) Only (III) above
(d) Both (I) and (II) above
(e) Both (II) and (III) above. **(1 mark)**

74. The Control Unit along with Main Memory and Arithmetic Logic Unit (ALU) is called as

(a) Input Unit
(b) Central Processing Unit
(c) Output Unit
(d) Digital computer
(e) Minicomputers. **(1 mark)**

75. The highest common factor of 64, 72 and 164 is

(a) 2
(b) 4
(c) 10
(d) 12
(e) 14. **(1 mark)**

76. The Arithmetic Logic Unit (ALU) contains one or more storage locations are called as

(a) Registers
(b) Card readers
(c) Magnetic disk drives
(d) Card punching machines
(e) Printers. **(1 mark)**

77. The appropriate average for a set of ratios, using the denominators of the ratio data as weights is

(a) Simple arithmetic mean
(b) Geometric mean
(c) Simple harmonic mean
(d) Weighted arithmetic mean
(e) Weighted harmonic mean. **(1 mark)**

78. A sum of Rs.1,800 deposited today in a bank gets tripled in a period of 6 years. Then, what is the annual rate of interest?

(a) 10.02%
(b) 15.26%
(c) 17.56%
(d) 18.26%
(e) 20.09%. **(2marks)**

END OF QUESTION PAPER

Suggested Answers
Quantitative Techniques (MSF1B1)

	ANSWER	REASON
1.	C	Let R_1 is the event of getting a red ball in the first attempt and R_2 is the event of getting a red ball in the second trial. As the two events are dependent, $P(R_1 \cap R_2) = P(R_1) \cdot P(R_2/R_1)$ $P(R_1) = \frac{3}{10}$, $P(R_2/R_1) = \frac{2}{9}$ $\therefore P(R_1 \cap R_2) = \frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$.
2.	D	The probability of the entire sample space S is 1. The probability of an event contained in the sample space should be greater than or equal to 0 and less than or equal to 1. If A and B are **independent** events, then the probability of (A and B) is equal to the product of probabilities of A and B. Hence option (d) is correct.
3.	C	$\sqrt[6]{a^{4b} x^6} \times (a^{\frac{2}{3}} x^8)^{-b} = \sqrt[6]{a^{4b} x^6} \times (a^{\frac{2}{3}} x^8)^{-b} = \left(a^{4b} x^6\right)^{\frac{1}{6}} \times (a^{\frac{2}{3}} x^8)^{-b}$ $= a^{\frac{4b}{6}} x\, a^{\frac{-2b}{3}} x^{-8b} = a^0 x^{1-8b} = x^{1-8b}$.
4.	D	$\log \frac{X+Y}{7} = \frac{1}{2}(\log X + \log Y)$ R.H.S = $\frac{1}{2}\log(XY) = \log(XY)^{1/2}$ $\Rightarrow \log \frac{X+Y}{7} = \log(XY)^{1/2}$ $\Rightarrow \frac{X+Y}{7} = (XY)^{1/2}$ Squaring on both sides $\Rightarrow (X+Y)^2 = 49XY$ $\Rightarrow X^2 + Y^2 + 2XY = 49XY$ $\Rightarrow X^2 + Y^2 = 47XY$ Dividing both sides by XY we get $\frac{X}{Y} + \frac{Y}{X} = 47.$
5.	A	H.M = 4, A.M = A, G.M = G We know that H.M= 4 = $\frac{G.M^2}{A.M} = \frac{G^2}{A}$ $= G^2 = 4A$ The relationship is $2A + G^2 = 27$. $2A + 4A = 27$

6A = 27

A = 27/6 = 9/2.

So $G^2 = 4(9/2) = 18$

Let the numbers be a, b

a + b = 9 ……..(1)

ab = 18

$(a - b)^2 = (a + b)^2 - 4ab = 81–72 = 9.$

$(a - b)^2 = 9$

$a - b = \pm 3$

let us take a – b = 3 …….(2)

by solving (1) and (2) we get a = 6, b = 3

similarly if we take a – b = – 3 we get a =3 and b = 6.

∴ The numbers are 3 and 6.

6. C

Let the numbers be $\frac{m}{r}$, m and mr.

The product of the numbers is $m^3 = 216$ or m = 6

The sum of the products of the pairs is $\frac{m}{r}$. m + m.mr + mr. $\frac{m}{r}$ = 156

$36(\frac{1}{r} + r + 1) = 156$

$3r^2 + 3r + 3 = 13r$

$3r^2 - 10r + 3 = 0$

$= 3r - 9r - r + 3 = 0$

(r – 3)(3r – 1) = 0

So r = 3 or 1/3.

If we take r =3 we get 2, 6, 18

If we take r =1/3 we get 18, 6, 2.

The highest number is 18.

7. B

There are 11 letters of 8 different kind C, (O, O), M, B, (I, I), (N, N), A, T.

In all the required combination some may contain all dissimilar letters, some may not contain all different letters. Following cases arises.

All the 4 letters are different

2 letters are alike, 2 are different

2 letters are alike of one kind, 2 letters are alike of second kind

There are 8 different letters. The required number of combination = $^8C_4$

There are 3 pairs of alike letters (O, O), (I, I), (N, N). One pair can be chosen in $^3C_1$ ways. Remaining 2 different can be selected from the remaining 7 different letters in $^7C_2$ ways. Hence the number of combinations of this type is $^3C_1 \times {}^7C_2$

Two pairs of similar letters can be chosen in $^3C_2$ ways.

Hence the total number of required combinations is $^8C_4 + (^3C_1 \times {}^7C_2) + {}^3C_2 = 136.$

8. B

If each observation in the data set increased by a constant 2.5 then the mean increased by that constant and standard deviation remains same.

Given mean = 12.6667, standard deviation = 6.18

If we increased a quantity 2.5 so the observations (5.5 + 9.5 + 14.5 + 17.5 + 20.5 + 23.5) / 6 =

15.1667.

Standard deviation =

$$\sqrt{\frac{(5.5-15.1667)^2+(9.5-15.1667)^2+(14.5-15.1667)^2+(17.5-15.1667)^2+(20.5-15.1667)^2+(23.5-15.1667)^2}{6}}$$

$= 6.18$

So mean is increased by 2.5 and standard deviation remains same.

Here mean is 15.1667 and standard deviation is 6.18.

9. B

Total number of ways in which 4 can be selected out of 30 = $^{30}C_4$

Number of ways in which 4 can be selected so that no women is included $= (30\text{-}5)_{C_4} = 25_{C_4}$

The number of ways in which 4 members can be selected so that at least one women is included is

$30_{C_4} - 25_{C_4} = 27405 - 12650 = 14755.$

10. C

The equation of line y = a + bx with a as intercept and b = slope

It passes through (1, 2) and (3, 4)

The intercept is a = $y' - \left[\frac{y'-y''}{x'-x''}\right]x' = 2 - \left[\frac{2-4}{1-3}\right]1 = 2-1 = 1.$

11. C

$$\frac{1}{2}x + \frac{1}{2}y - \frac{1}{6}z = 5 \text{.....(i)}$$

$$\frac{1}{3}x + \frac{1}{4}y + \frac{1}{2}z = 10 \text{.....(ii)}$$

$$\frac{1}{6}x + \frac{1}{4}y + \frac{1}{3}z = 7 \text{.....(iii)}$$

By doing (ii) + (iii) − (i) we get

$$\left(\frac{1}{3}+\frac{1}{2}+\frac{1}{6}\right)z = (7+10-5)$$

Or, $z = 12.$

12. B

$$^{n}c_6 : {}^{(n-3)}c_3 = 33:4$$

$$\text{Or, } \frac{^{n}c_6}{^{(n-3)}c_3} = \frac{33}{4}$$

$$\text{Or, } 4\times {}^{n}c_6 = 33\times {}^{(n-3)}c_3$$

$$\text{Or, } 4\times\frac{n!}{6!(n-6)!} = 33\times\frac{(n-3)!}{3!(n-3-3)!} = 33\times\frac{(n-3)!}{3!(n-6)!}$$

$$\text{Or, } 4\times\frac{(n-2)(n-1)n}{4\times5\times6} = 33$$

$$\text{Or, } (n-2)(n-1)n = 30\times33 = 9\times10\times11 = (11-2)(11-1)(11)$$

So the value of n =11.

13. E

The quadratic equation is $x^2 + 7x + 12 = 0$

$$x = \frac{(-7)\pm\sqrt{7^2-4(1)(12)}}{2(1)} = \frac{-7\pm1}{2} = -3 \text{ and } -4.$$

14. E

$16Q^2 - 81 = (4Q)^2 - (9)^2 = (4Q - 9)(4Q + 9)$ (Q $a^2 - b^2 = (a + b)(a - b)$)

15. A

$$^{8}P_6 = \frac{8!}{(8-6)!} = \frac{8!}{2!} = 20{,}160.$$

16. B

Given $x^y = \log_e x$

Taking logarithms on both sides w.r.t to base e

$y \log_e x = \log_e(\log_e x)$

Differentiating both sides w.r.t x

$$y'(\log_e x) + \frac{y}{x} = \frac{1}{(\log_e x)x}$$

At x = e, $y' + \frac{y}{e} = \frac{1}{e} = y' = \frac{1-y}{e}$.

17. A

$$\lim_{x\to -1} \frac{2x^2 - x - 3}{x^2 - 2x - 3} = \lim_{x\to -1} \frac{2x^2 - 3x + 2x - 3}{x^2 - 3x + x - 3}$$

$$= \lim_{x\to -1} \frac{x(2x-3) + 1(2x-3)}{x(x-3) + 1(x-3)}$$

$$= \lim_{x\to -1} \frac{(2x-3)(x+1)}{(x-3)(x+1)}$$

$$= \lim_{x\to -1} \frac{(2x-3)}{(x-3)}$$

$$= \frac{2(-1)-3}{-1-3} = \frac{-5}{-4} = \frac{5}{4}.$$

18. B

$$TC = \frac{10^2}{Q} + \frac{2.25Q}{2}$$

$$\frac{d(TC)}{dQ} = \frac{-10^2}{Q^2} + \frac{2.25}{2}$$

Equating $\frac{d(TC)}{dQ} = 0$ to we have $\frac{-10^2}{Q^2} + \frac{2.25}{2} = 0$

$$Q^2 = \frac{2 \times 10^2}{2.25} = 88.8889$$

$Q = \sqrt{88.8889} = 9.4281.$

19. B

The following are true:

- Interpolation and Extrapolation provide us with only estimates of the **dependent** variable.
- For interpolation and extrapolation to work there must exist a functional relationship between an independent variable and a dependent variable and it must be required to estimate the value of the dependent variable corresponding to a certain value of independent variable.
- Interpolation is a statistical technique, which through a study of time series of known figures of population, allows us to make insertions.
- Extrapolation allows to forecast or anticipate a value for some future data.

In interpolation or extrapolation population is taken to be a function of time or years and while time is the independent variable, population is the dependent variable.

Hence option (b) is correct.

20. D

The given option (d), The tallest rectangle represents the class interval with the lowest frequency is false and the correct one is the tallest rectangle represents the class interval with highest frequency.

So, option (d) is the correct answer.

21. E Let the IRR of the project be 'r'

$$\therefore \quad 35 = \frac{15}{(1+r)^1} + \frac{10}{(1+r)^2} + \frac{10}{(1+r)^3} + \frac{20}{(1+r)^4}$$

For, r = 20%,

$$\text{RHS} = \frac{15}{1.20} + \frac{10}{(1.20)^2} + \frac{10}{(1.20)^3} + \frac{20}{(1.20)^4} = 34.8765$$

For, r = 19%,

$$\text{RHS} = \frac{15}{1.19} + \frac{10}{(1.19)^2} + \frac{10}{(1.19)^3} + \frac{20}{(1.19)^4}$$

$= 35.5742$

$\therefore$ The following pairs of 'r' and values of RHS are obtained:

r = 19% RHS = 35.5742

r = ? RHS = 35

r = 20% RHS = 34.8765

$$r = 19 + \frac{35.5742 - 35}{35.5742 - 34.8765} \Rightarrow 19.822\%.$$

22. C A graph of cumulative frequency distribution is called as ogive.

23. A A relative frequency polygon indicates the skewness of the distribution.

24. E A positively skewed distribution curve tails off towards the higher end and for such a curve

Mean > Median > Mode.

25. E The sum of deviations of the items from the arithmetic mean is always zero.

26. C

P(A's solving the problem) = $\frac{2}{5}$

P(B's solving the problem) = $\frac{5}{11}$

P(A's not solving the problem) = $1 - \frac{2}{5} = \frac{3}{5}$

P(B's not solving the problem) = $1 - \frac{5}{11} = \frac{6}{11}$

P(Both A and B are unable to solve the problem) = $\frac{3}{5} \times \frac{6}{11} = \frac{18}{55}$

P(The problem being solved) = $1 - \frac{18}{55} = \frac{37}{55} = 0.673$.

27. B Let E_1 be the event that be the event the ball is drawn from bag A. E_2 the event that it is drawn from bag B and E that the ball is red.

We have to find $P(E_2/E)$

Since both the bags are equally likely to be selected, we have

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Also, $P(E|E_1) = \frac{3}{5}$ and $P(E|E_2) = \frac{5}{9}$

Hence by Bays' theorem, we have

$$P(E_2|E) = \frac{P(E_2).P(E|E_2)}{P(E_1).P(E|E_1)+P(E_2)P(E|E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{9}}{\left(\frac{1}{2}\times\frac{3}{5}\right)+\left(\frac{1}{2}\times\frac{5}{9}\right)} = \frac{25}{52}.$$

28. E The probability that a boy will get a scholarship = 0.9
The probability that a girl will get a scholarship = 0.8
The probability at least one of them will get the scholarship is
P(A or B) = P(A) + P(B) – P(A and B)
= 0.9 + 0.8 – (0.9 × 0.8) = 1.7 – 0.72 = 0.98.

29. A Time taken in covering 25 miles at a speed of 30 mph = 50 minutes.
Time taken in covering 50 miles at a speed of 40 mph = 75 minutes
Distance covered in 6 minutes at a speed of 10 mph = 1 mile
Time taken in covering 24 miles at a speed of 24 mph = 60 minutes
Therefore, taking the time taken as weights we have the weighted mean as

Speed (in mph) [x]	Time taken (in minutes) [W]	Wx
30	50	1500
40	75	3000
10	6	60
24	60	1440
	ΣW = 191	ΣWx = 6000

∴ Average speed = $\frac{6000}{191} = 31.41$ mph.

30. D

Income arranged in ascending order	No. of persons	c.f	Income arranged in ascending order	No. of persons	c.f
800	16	16	1800	30	96
1,000	24	40	2000	20	116
1,500	26	66	2500	6	122

Median = Size of $\left(\frac{N+1}{2}\right)^{th}$ item = $\frac{122+1}{2} = 61.5^{th}$ item

∴ Size of 61.5^{th} item = Rs.1500.

31. D Let P(A) and P(B) denote the percentage of city population who read newspapers A and B. Then from given data we have

P(A) = 25% = $\frac{1}{4}$; P(B) = 20% = $\frac{1}{5}$; P(A and B) = 8% = $\frac{2}{25}$

∴ Percentage of those who read A but not B = 0.25 – 0.08 = 0.17

= $P(A \text{ and } \bar{B})$ = P(A) – P(A and B) = $\frac{1}{4} - \frac{2}{25} = \frac{17}{100} = 17\%$

Similarly $P(\bar{A} \text{ and } B)$ = P(B) – P(A and B) = 0.20 – 0.08 = 0.12

= $\frac{1}{5} - \frac{2}{25} = \frac{3}{25} = 12\%$

If P(C) denote the percentage of those who look into advertisement then from the given data we obtain

P(C) = 30% of $P(A \text{ and } \bar{B})$ + 40% of $P(\bar{A} \text{ and } B)$ + 50% of P(A and B)

= 0.3 × 0.17 + 0.4 × 0.12 + 0.5 × 0.08 = 0.139 = 13.9%.

$$= \frac{3}{10}\times\frac{17}{100}+\frac{2}{5}\times\frac{3}{25}+\frac{1}{2}\times\frac{2}{25}=\frac{51+48+40}{1000}=\frac{139}{1000}$$

$\therefore$ Thus the percentage of population who read an advertisement is 13.9%.

32. C $f(x) = x^3 - 8x^2 + 21x - 18$

The function will be maximum or minimum where $\frac{dy}{dx} = 0$

$$f'(x) = \frac{dy}{dx} = 3x^2 - 16x + 21 = 0$$

x = 3 and x =7/3

$f''(x) = 6x - 16$

The condition for the minimum value is that second derivative $f''(x) > 0$

At x =3 , $f''(x) = 6(3) - 16 = 2 > 0$

At x =7/3 $f''(x) = 6(7/3) - 16 = -2 < 0$

So at x =3 $f''(x) > 0$

The function f(x) is minimum at x =3.

33. A A derivative is a slope of a function if $f'(x) > 0$ and $f''(x) > 0$ then function is increasing at increasing rate.

34. A Mean is a mathematical average. So it is most affected by the presence of extreme values all other are positional measures, so they are not effected by the presence of extreme values.

35. A Given that,

$$u = \log(x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2),$$

$$\text{or}, u = \log f(x, y, z)$$

$$\text{Now}, u_x = \frac{4x^3 - 2xy^2 - 2z^2x}{f(x, y, z)}, u_y = \frac{4y^3 - 2x^2y - 2yz^2}{f(x, y, z)},$$

$$u_z = \frac{4z^3 - 2y^2z - 2zx^2}{f(x, y, z)}$$

$$\therefore xu_x + yu_y + zu_z = \frac{4[x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2]}{f(x, y, z)}$$

$$= \frac{4f(x, y, z)}{f(x, y, z)} = 4.$$

36. C Here highest frequency is 150 so mode lies in the class (25 – 30)

$$M_o = L_{mo} + \frac{f - f_1}{2f - f_1 - f_2} \times W$$

f = 150, f_1 = frequency of preceding class =110, f_2 = frequency of succeeding class = 120

Where L_{mo} = 25; W =5

$$M_0 = = 25 + \frac{150 - 110}{2(150) - 110 - 120} = 25 + \frac{40}{70} \times 5$$

= 25 + 2.8571 = 27.8571.

37. D Calculation of harmonic mean

X	1/X
25	0.04
45	0.02222
75	0.01333

5	0.2
10	0.1
20	0.05
50	0.02
60	0.01667
290	0.4622

$$\therefore \text{H.M} = \frac{N}{\Sigma\left(\frac{1}{X}\right)} = \frac{8}{0.4622} = 17.3085.$$

38. D

The value of standard deviation remains the same if in series each of the observation is increased or decreased by a constant quantity. Thus, the value of variance remains the same if every observation is increased by 3.

39. E

The derivative of a function, y = f(x) at x = a is equal to $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$. It indicates the rate at which f(x) is changing with respect to the corresponding change in x.

40. C

$$\text{Standard deviation} = \sqrt{\frac{\sum x^2}{n-1} - \frac{n\bar{x}^2}{n-1}} = \sqrt{\frac{385}{10-1} - \frac{10(5.5)^2}{10-1}} = \sqrt{9.1667} = 3.0277.$$

41. C

Percentage of dividend declared	Number of companies	Cumulative frequency
10 – 20	6	6
20 – 30	15	21
30 – 40	10	31
40 – 50	14	45
50 – 60	11	56

Here N = 56, 3N/4 = 42

L_q = Lower limit of the quartile class = 40

F = Cumulative frequency up to quartile class = 31

f_q = Frequency of the quartile class = 14

W = Width of the class interval = 10.

$$Q_3 = L_q + \left[\frac{\frac{3N}{4} - F}{f_q}\right] \times W$$

$$= 40 + \left[\frac{42 - 31}{14}\right] \times 10 = 47.8571.$$

42. A

Let the events be denoted as follows:

A: Reader of the magazine is a male and B: Reader of the magazine is more than 35 years of age

Now, we are given that,

$P(A \text{ and } B) = 0.20,\ P(A \text{ and } \bar{B}) = 0.40$ and $P(\bar{B}) = 0.70 \Rightarrow P(B) = 0.30$

P(A) = P(A and B) + P(A and $\bar{B}$) = 0.2 + 0.4 = 0.60.

The probability that a randomly selected male subscriber is under 35 years is given by:

$$P(\bar{B}/A) = \frac{P(A \text{ and } \bar{B})}{P(A)} = \frac{0.40}{0.60} = \frac{2}{3} = 0.67.$$

43. C

For set 1, $\mu_1 = 8.333$ and $\sigma_1 = 3.859$

For set 2, $\mu_2 = 5$ and $\sigma_2 = 2.915$

(Consider these 2 sets of observations as populations).

Combined mean $\mu = \frac{N_1 \mu_1 + N_2 \mu_2}{N_1 + N_2} = \frac{(3 \times 8.333) + (4 \times 5)}{3 + 4} = 6.429$

$d_1 = \mu_1 - \mu = 8.333 - 6.429 = 1.904$

$d_2 = \mu_2 - \mu = 5 - 6.429 = -1.429$

σ [i.e. standard deviation for the combined set of 7 observations]

$$= \sqrt{\frac{(3 \times 3.859^2) + (4 \times 2.915^2) + (3 \times 1.904^2) + [4 \times (-1.429)^2]}{3 + 4}} = 3.7360.$$

44. D The dual formulation can be solved for optimal solution. All other statements are true.

45. D The profit function Z = 2x+3y

The vertices are (1, 0), (6, 0), (4, 9) and (0, 15).

For (1, 0) Z = 2(1) + 3(0) = 2 + 0 = 2

For (6, 0) Z = 2(6) + 3(0) = 12 + 0 = 12

For (4, 9) Z = 2(4) + 3(9) = 8 + 27 = 35

For (0, 15) Z = 2(0) + 3(15) = 0 + 45 = 45.

So for (0, 15) the profit function is maximum

46. E If a objective function in a graphical linear programming problem is parallel to an edge of the feasible region. It mean there is more than on multiple number of optimal solution.

47. D Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to constraints

$x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_3 \leq 420$

$x_1, x_2, x_3 \geq 0.$

For applying simplex method, we convert the inequalities into equalities by adding the slack variables S_1, S_2 and S_3 for the constraints.

The problem now is: Maximize $Z = 3x_1 + 2x_2 + 5x_3 + 0S_1 + 0S_2 + 0S_3$

Subject to: $x_1 + 2x_2 + x_3 + S_1 = 430$

$3x_1 + 2x_3 + S_2 = 460$

$x_1 + 4x_3 + S_3 = 420$

$x_1, x_2, x_3 \geq 0, S_1 \geq 0, S_2 \geq 0, S_3 \geq 0$

The tableaus are constructed below to obtain the optimal solution. The pivots identified in each tableau are circled.

Tableau 1

Profit			3	2	5	0	0	0	Min Xb/xk
Variables		x_b	x_1	x_2	x_3	S_1	S_2	S_3	
Profit	Variables	Solution							
0	S_1	430	1	2	1	1	0	0	430/1 = 430
0	S_2	460	3	0	2	0	1	0	460/2 = 230
0	S_3	420	1	0	4	0	0	1	420/4 = 105
	$Z_j - C_j$	0	–3	–2	–5	0	0	0	

Tableau 2

Profit			3	2	5	0	0	0	
Variables			x_1	x_2	x_3	S_1	S_2	S_3	Min Xb/xk
Profit	Variables	Solution							
0	S_1	325	$\frac{3}{4}$	2	0	1	0	$-\frac{1}{4}$	= 325/2
0	S_2	250	$\frac{5}{2}$	0	0	0	1	$-\frac{1}{2}$	-
5	x_3	105	$\frac{1}{4}$	0	1	0	0	$\frac{1}{4}$	-
	$Z_j - C_j$		$\frac{-7}{4}$	–2	0	0	0	$\frac{5}{4}$	

Tableau 3

Profit			3	2	5	0	0	0	Min Xb/xk
Variables			x_1	x_2	x_3	S_1	S_2	S_3	
Profit	Variables	Solution							
2	x_2	325/2	$\frac{3}{8}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{8}$	= 433.333
0	S_2	250	$\frac{5}{2}$	0	0	0	1	$-\frac{1}{2}$	100
5	x_3	105	$\frac{1}{4}$	0	1	0	0	$\frac{1}{4}$	420
	$Z_j - C_j$		–1	0	0	1	0	1	

Tableau 4

Profit			3	2	5	0	0	0
Variables			x_1	x_2	x_3	S_1	S_2	S_3
Profit	Variables	Solution						
2	x_2	125	0	1	0	$\frac{1}{2}$	$-\frac{3}{20}$	$\frac{-1}{20}$
3	x_1	100	1	0	0	0	$\frac{2}{5}$	$-\frac{1}{5}$
5	x_3	80	0	0	1	0	$-\frac{1}{10}$	$\frac{3}{10}$

$Z_j - C_j$	0	0	0	1	$\frac{2}{5}$	$\frac{4}{5}$

The optimum solution is $x_2 = 125$ and $x_3 = 80$ and $x_1 = 100$ maximize $Z = 950$.

48. D The given data can be formulated in appropriate mathematical form as follows

Maximize $Z = 5x_1 + 8x_2$

Subject to constraints:

$15x_1 + 10x_2 \leq 180$

$10x_1 + 20x_2 \leq 200$

$15x_1 + 30x_2 \leq 210$

$x_1, x_2 \geq 0$

Where x_1, x_2 are decision variables.

Considering the constraints as equations

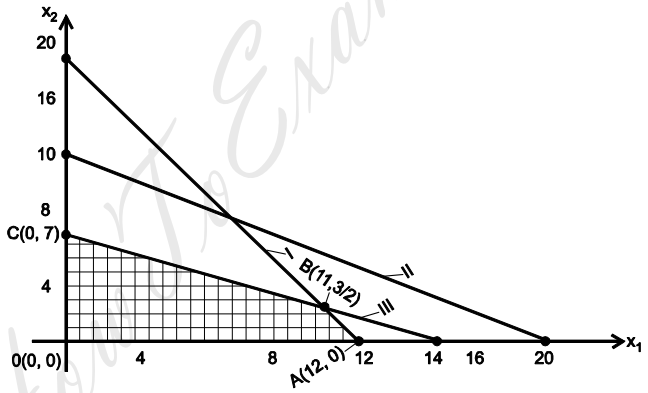
$15x_1 + 10x_2 = 180$……..(I)

$10x_1 + 20x_2 = 200$……..(II)

$15x_1 + 30x_2 = 210$……..(III)

We get the points by putting $x_1 = 0$ and $x_2 = 0$ in eq(I),(II) and (III) we get the ordered pairs as (0, 18), (12, 0), (0, 10), (20, 0) and (0, 7), (14, 0)

The feasible region is shown in the graph below



The corner point of the solution space are

O = (0, 0)

A = (12, 0)

B = (11, 3/2)

C = (0, 7)

The value of the objective function at these corner points can be determined.

Corner points	Coordinates of corner points (x_1, x_2)	Objective function $Z = 5x_1 + 8x_2$	Value
O	(0, 0)	5(0) + 8(0)	0
A	(12, 0)	5(12) + 8(0)	60
B	(11, 3/2)	5(11) + 8(3/2)	67
C	(0, 7)	5(0) + 8(7)	56

The maximum value of Z is 67 found at corner point B i.e., $x_1 = 11$, $x_2 = 3/2$

At B(11, 3/2) the optimum solution is reached.

49. C The reciprocal of the terms in a harmonic progression are in arithmetic progression. And vice versa. Therefore the reciprocals of a H.P. cannot be in H.P. There is no such connection between the H.P. and the G.P. If the terms in the corresponding A.P. are in the increasing order then the terms of the H.P. will be in the decreasing order and vice versa. Hence there is no reason why

terms of a H.P. will always be in increasing or decreasing order.

50. D $y = f(x) = 7x - 5$

Solving x in terms of y

$x = f^{-1}(y) = \frac{y+5}{7}$ is the inverse.

51. A The dual of lpp is

Maximize $Z^* = 5y_1 + 6y_2 + 8y_3$

Subject to constraints:

$y_1 + 3y_2 + 6y_3 \leq 2$

$2y_1 + 4y_2 + 7y_3 \leq 3$

$3y_1 + 5y_2 + 8y_3 \leq 4$

And $y_1, y_2, y_3 \geq 0$

52. E

$$xyz = \frac{\sqrt[9]{81} \times \sqrt[7]{49}}{\sqrt[3]{25}} \times \frac{\sqrt[9]{625} \times \sqrt[3]{3}}{\sqrt[7]{7^3}} \times \frac{\sqrt[9]{9}}{\sqrt[9]{5^7} \times \sqrt[7]{7^6}}$$

$$= \frac{3^{\frac{4}{9}} \times 7^{\frac{2}{7}}}{5^{\frac{2}{3}}} \times \frac{5^{\frac{4}{9}} \times 3^{\frac{1}{3}}}{7^{\frac{3}{7}}} \times \frac{3^{\frac{2}{9}}}{5^{\frac{7}{9}} \times 7^{\frac{6}{7}}}$$

$$= \frac{3^{\left(\frac{4}{9}+\frac{3}{9}+\frac{2}{9}\right)}}{5^{\left(\frac{2}{3}-\frac{4}{9}+\frac{7}{9}\right)} \times 7^{\left(\frac{3}{7}+\frac{6}{7}-\frac{2}{7}\right)}}$$

$$= \frac{3^{\frac{9}{9}}}{5^{\frac{9}{9}} \times 7^{\frac{7}{7}}} = \frac{3}{5 \times 7} = \frac{3}{35}.$$

53. D Three terms are in geometric progression and the second and the third terms are 16 and 64 respectively

a, ar , ar^2 = a , 16, 64

$$\frac{ar^2}{ar} = \frac{64}{16} = 4$$

ar = 16

a(4) = 16

a = 4

So first term = 4.

54. D If mean = 29.51 and median = 39.69, then

Mode =3median – 2mean =3(39.69) – 2(29.51) = 60.05.

55. A $\log_3 81 = \log_3 3^4 = 4\log_3 3 = 4.1 = 4 (Q \log_3 3 = 1)$.

56. A Given that,

$$f(x, y, z) = x^2(y - z) + y^2(z - x) + z^2(x - y)$$

$$\frac{\partial f}{\partial x} = f_x = 2x(y - z) - y^2 + z^2$$

$$\frac{\partial f}{\partial y} = f_y = x^2 + 2y(z - x) - z^2$$

$$\frac{\partial f}{\partial z} = f_z = -x^2 + y^2 + 2z(x - y)$$

$$\therefore f_x + f_y + f_z = 0.$$

57. C

Given that $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$$\therefore \frac{(m/2)[2a+(m-1)d]}{(n/2)[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\therefore \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$$2an + mnd - nd = 2am + mnd - md$$

$$2a(n-m) = (n-m)d$$

$$= d = 2a.$$

Now $\frac{T_m}{T_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)2a}{a+(n-1)2a} = \frac{2m-1}{2n-1}$.

58. D

According to the Bienayme-Chebyshev rule, at least 75 percent of the observations in a sample are contained within two standard deviations around the mean, regardless of the shape of the distribution.

59. B

Arranging the data in the ascending order

11, 15, 15, 17, 19, 20, 21, 22, 23, 25, 29. Here N is odd (N+1)/2 term is the median

Median = (N+1)/2 = 12/2 = 6 th item = 20.

60. A

$$\text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100$$

$$= \frac{\sqrt{\text{Variance}}}{\text{Mean}} \times 100$$

$$\text{or} \quad 20 = \frac{\sqrt{16}}{\text{Mean}} \times 100$$

$$\text{or} \quad \text{Mean} = \frac{4}{20} \times 100 = 20.$$

61. D

Range = Highest value – Lowest value = 20 – 7 = 13.

62. D

Class	Mid-value (m)	Frequency (f)	f × m
20 – 25	22.5	3	67.5
25 – 30	27.5	16	440
30 – 35	32.5	22	715
35 – 40	37.5	18	675
40 – 45	42.5	14	595
45 – 50	47.5	10	475
50 – 55	52.5	7	367.5
55 – 60	57.5	6	345
60 – 65	62.5	4	250
	Total	100	3,930

Mean = $\frac{\Sigma fm}{\Sigma f} = \frac{3930}{100}$ = 39.3 years.

63. B

N_1 = number of males N_2 = number of females

So that $N_1 + N_2 = 100$

We are given $\bar{X}_{12} = 5000$; $\bar{X}_1 = 6000$; $\bar{X}_2 = 4000$

Substituting the value in the formula

$$\bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$$

$$\Rightarrow \quad 5000 = \frac{N_1(6,000) + N_2(4000)}{100}$$

$$\Rightarrow \quad 5,00,000 = 6000N_1 + [(100 - N_1)\,4000]$$

[Since $N_1 + N_2 = 100$; $N_2 = 100 - N_1$)]

$$\Rightarrow \quad 5,00,000 = 6000N_1 + 4,00,000 - 4000N_1$$

$$\Rightarrow \quad 2000\,N_1 = 1,00,000$$

$$\Rightarrow \quad N_1 = 50 \text{ and } N_2 = 50.$$

64. B Let the numbers be a, b. Let the A.M be A, G.M be G and H.M be H

As A.M. exceeds G.M by 2. So, A – G = 2; Or, $(A-2)^2 = G^2$ ……..(i)

As G.M exceeds H.M by 8/5. So, G – H = 8/5 Or, A – 2 – H = 8/5

Or, $A - H = \frac{18}{5}$

$G^2 = AH = A(A - \frac{18}{5})$ Putting this value of G^2 in equation (i) we get

$(A-2)^2 = A(A - \frac{18}{5})$ Or, $-4A + 4 = -\frac{18}{5}A$

Or, 2A = 20 Or, A = 10. Or, (a+b) = 20

So, G = 8 Or, $\sqrt{ab} = 8$ i.e. ab = 64

So, $a + \frac{64}{a} = 20$

Or, $a^2 - 20a + 64 = 0$.

Or, $a^2 - 16a - 4a + 64 = 0$

Or, a(a – 16) – 4(a – 16) = 0

Or, (a – 4)(a – 16) = 0

So the vale of the numbers would be (4, 16) or (16, 4)

So the highest number is 16.

65. A For any function y = f(x), if $f'(x) = 0$ at x = a, the function is said to have a relative maxima or minima.

66. E High-level languages are English like languages with a predefined format and machine independent and can be used on various computer systems. Option (e) is correct.

67. A In a linear equation Y = a + bX , a and b represent Y- intercept and slope of the curve respectively where X and Y represent independent and dependent variables respectively. Thus if the slope and intercept of a line on Y-axis 4 and 3 respectively, then the equation of the line is
Y = 3 + 4X.

68. E The limiting value of a function, y = f(x) at x = a is defined as the value to which y approaches, as x approaches a; it is neither the minimum value, nor the maximum value.

69. E For any real number, a, the interval [a, + ∝) indicates the set of all real numbers greater than or equal to a, it includes all the even and odd numbers greater than or equal to a.

70. D The symbol  indicates off-page connector in flow chart.

71. E Packet is a bundle of data. Baud is the rate at which the medium can transfer groups of data. All other statements are true.

72. E The word size in mini computers is 32 bits or more.

73. A In digital computers uses the properties of electronic circuits to represent exactly two states – on and off.

74. B The Control Unit along with Main Memory and Arithmetic Logic Unit is called as Central Processing Unit

75. B First we consider 64 and 72. The HCF for these two quantities is calculated as follows:

64) 72 (1
64
8)64(8
64
0

The HCF is 8, now consider 8 and 164 and obtain the HCF for these two quantities.

8) 164 (20
160
4)8(2
8
0

The highest common factor is 4 for the three quantities.

76. A The Arithmetic Logic Unit (ALU) contains one or more storage locations are called as registers.

77. D The appropriate average for a set of ratios using the denominators of the ratio data as weights is weighted arithmetic mean.

78. E Let r be the rate of interest

P_n = The value at the end of period n, P_0 = The value at the beginning of period n

$P_n = P_0(1+ r)^n$

$5400 = 1800(1+ r)^6$

$(1+ r)^6 = 5400/1800 = 3$

$(1+ r) = 3^{1/6} = 1.20098$

$r = 1.20098 – 1 = 0.20098 = 20.098\% = 20.09\%$.