B4.2-R3: DISCRETE STRUCTURES

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Determine $\{\phi\} \cap \{a, d, \{\phi\}\}$.

b) Obtain an equivalent formula for $p \wedge (q \Leftrightarrow r)$ containing neither the biconditional nor the conditional.

c) For any Boolean Algebra$(B,+,\bullet,/)$ or $(B,\vee,\wedge,/)$, $\forall a \in B$, show that $(a')' = a$.

d) If the binary operation $o: R \times R \rightarrow R$ is defined by $a \, o \, b = a + b - 5$, prove that o operation has an identity. Also find inverse of an element $a \in R$ with respect to this binary operation.

e) Prove that the product of r consecutive integers is divisible by r!.

f) If $G = (V,E)$ where $V = \{v_1, v_2, v_3, v_4, v_5, v_6)$
and $E = \{v_1 v_2, v_1v_5, v_1v_6, v_2v_3, v_2v_5, v_3v_4, v_4v_5\}$
Then find complement of G.

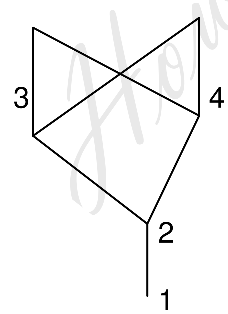
g) Represent the following set by regular expression:
$(\mathbf{a}^n \mid \mathbf{n}$ is divisible by 2 or 3 or $n = 5)$

(7x4)

2.

a) If R_1 and R_2 are two equivalence relations on a set A, then prove that $R_1 \cap R_2$ is also an equivalence relation on A. What can you say for $R_1 \cup R_2$?

b) Hasse diagram of poset S = {1,2,3,4,5,6} is given below. If A={2,3,4} is a subset of S, find upper bounds, lower bounds, supremum and infimum of A.



c) Let N be the set of natural numbers including zero. Determine which of the following functions are one to one, which are onto and which are one to one onto?

i) $f: N \rightarrow N \quad f(j) = j^2 + 2$

ii) $f: N \rightarrow N \quad f(j) = 1$ if j is odd
$= 0$ otherwise

(6+6+6)

3.

a) Without using truth table show that
$((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R))$ is a tautology.

b) Check the validity of the following argument:
If the rents of Hotels in Bombay are fixed or the prices of the commodities are reduced, then the income of businessmen shall decrease. If the income of businessmen decrease then the farmers shall feel happy. The farmers never feel happy. Therefore the rents of the hotels are not fixed.

c) Show that the operations +,-, and . on Z_m are well defined functions.

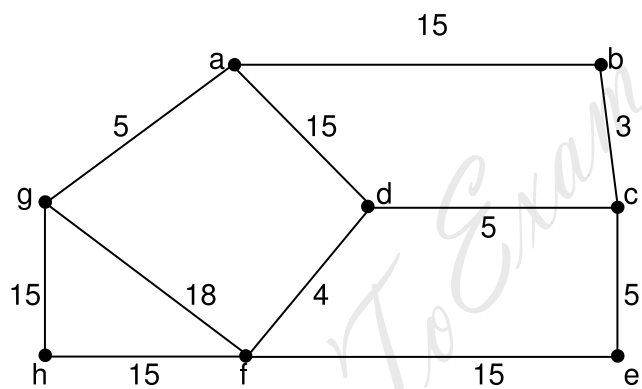
(6+6+6)

4.

a) Simplify the following expressions in a Boolean algebra.
   i) $(a+b) \bullet a' \bullet b'$
   ii) $(a + a' \bullet b) \bullet (a' + a \bullet b)$

b) Show that the set P_n of all permutations on the set $A=\{a_1,a_2,a_3,\ldots.a_n\}$ is a finite group of order n! with respect to the composition of "product of permutations or composition of permutations".

c) Determine a railway network of minimal cost for the cities in the figure:

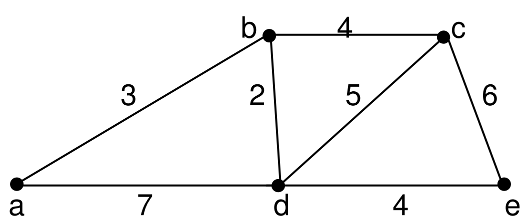


(6+6+6)

5.

a) How many vertices will the following graphs have if they contain:
   i) 16 edges and all vertices of degree 2.
   ii) 21 edges, 3 vertices of degree 4, and the other vertices of degree 3.
   iii) 24 edges and all the vertices of the same degree.

b) Apply Dijkstra's algorithm to find shortest paths from the source node a to all other nodes in the following graph:

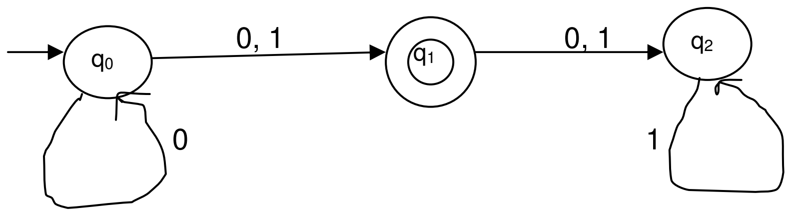


c) Show that any graph with 4 or fewer vertices is planar.

(6+6+6)

6.

a) Convert the non deterministic finite automata in the following graph into equivalent deterministic machine:



b) Let Σ={a,b} and let $n_a(w)$ and $n_b(w)$ denote the number of a's and number of b's in the string w respectively. Then write a grammar to generate the language L.
L ={w: $n_a(w) = n_b(w)$}

c) Show that the language
L={$a^n b^k c^{n+k}$: n $\geq$0, k$\geq$0} is non regular.

(6+6+6)

7.

a) Find a generating function for a_r = the number of ways the sum r can be obtained when:
   i) 2 distinguishable dice are tossed.
   ii) 2 distinguishable dice are tossed and the first shows an even number and the second shows an odd number.

b) Solve the following recurrence relation by substitution:
$a_n = a_{n-1} + 1/n(n+1)$ where $a_0 = 1$

c) Prove that in a connected (simple) plane graph G, with |E| >1
   i) |E| $\leq$ 3|V|-6
   ii) there exists a vertex of G such that degree(v) $\leq$5

(6+6+6)