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07MCA11B

First Semester MCA Degree Examination, Dec. 07 / Jan. 08
Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least TWO questions from each part.***

PART – A

1 a. If $\sec A + \tan A = a$, prove that $\sin A = \dfrac{a^2-1}{a^2+1}$ **(07 Marks)**

b. If $A+B=45^0$, then show that $(1+\tan A)(1+\tan B)=2$. Hence deduce the value of $\tan^{-1} 22\frac{1}{2}^0$. **(07 Marks)**

c. With the usual notations, prove that $\sin(A+B)=\sin A \cos B + \cos A \sin B$. **(06 Marks)**

2 a. Prove that $(1+\cos\theta+i\sin\theta)^n + (1+\cos\theta-i\sin\theta)^n = 2^{n+1}.\cos^n \theta/2.\cos\left(\dfrac{n\theta}{2}\right)$. **(07 Marks)**

b. Prove that $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$. **(07 Marks)**

c. Using De'Moivre's theorem, solve the equation $x^4 - x^3 + x^2 - x + 1 = 0$. **(06 Marks)**

3 a. If $A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ compute AB and BA. Show that $AB \neq BA$. **(07 Marks)**

b. Express the matrix A as the sum of a symmetric and a skew–symmetric matrix.

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}.$$ **(07 Marks)**

c. Find the rank of matrix

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$ **(06 Marks)**

4 a. Test for consistency and hence solve

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$ **(07 Marks)**

b. Find the eigen values and eigen Vectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$ **(07 Marks)**

c. Using Cayley – Hamilton theorem, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}.$$ **(06 Marks)**

PART – B

5 a. If $y = \cos x \ \cos 2x \ \cos 3x$, find y_n. **(07 Marks)**

b. If $y = \dfrac{1}{4x^2 + 8x + 3}$, find y_n. **(07 Marks)**

c. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**

6 a. Find the pedal equation of the curve $\dfrac{2a}{r} = 1 - \cos\theta$. **(07 Marks)**

b. Find the angle between the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$. **(07 Marks)**

c. Evaluate $\lim\limits_{x \to 0} \dfrac{x\cos x - \log(1+x)}{x^2}$. **(06 Marks)**

7 a. Evaluate $\int \dfrac{2x+3}{x^2 + x - 30} dx$. **(07 Marks)**

b. Evaluate $\int \dfrac{dx}{\sqrt{1 + x - x^2}}$. **(07 Marks)**

c. Evaluate $\int_0^{\pi/2} \dfrac{\cos x}{1 + \sin^2 x} dx$. **(06 Marks)**

8 a. Solve $\dfrac{dy}{dx} = \dfrac{x(2\log x + 1)}{\sin y + y\cos y}$. **(07 Marks)**

b. Solve $(x^2 - y^2)dx = xydy$. **(07 Marks)**

c. Solve $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$. **(06 Marks)**

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