MAT11

NEW SCHEME

USN 1MS04ME087

First Semester B.E Degree Examination, February/March 2005

Common to all Branches

Engineering Mathematics - I

Time: 3 hrs.] [Max.Marks : 100

Note: ***1. Answer FIVE full questions, choosing at least ONE question from EACH PART.***
2. All questions carry equal marks.

PART - A

1. (a) Show that the lines whose direction cosines satisfy the relations $l+m+4n=0$ and $mn+nl+lm=0$ are parallel. **(6 Marks)**

(b) Derive the equation of the plane in the intercept form. Also find the equation of the plane having y - intercept 10, z - intercept 4 and perpendicular to the plane $7x+y+13z-17=0$. **(4+3=7 Marks)**

(c) Find the image of the point (1, -1, 2) in the plane $2x+2y+z=11$. **(7 Marks)**

2. (a) Show that the lines $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$ and $x+2y+3z-8=0=2x+3y+4z-11$ intersect. Also find their point of intersection. **(6 Marks)**

(b) Find the coordinates of the point of intersection of the line of S.D with the lines

$\frac{x+3}{2}=\frac{y-6}{3}=\frac{z-3}{-2}$ and $\frac{x}{2}=\frac{y-6}{2}=\frac{z}{-1}$ and hence find the shortest distance. **(7 Marks)**

(c) Find the equation of the right circular cone with vertex (2, -3, -4), semivertical angle 30^0 and whose axis is equally inclined to the coordinate axes. **(7 Marks)**

PART - B

3. (a) If $y=(x^2-1)^n$ show that y_n satisfies the equation:

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0.$$ **(6 Marks)**

(b) Establish the pedal equation of the polar curve :

$r^n=a^n \sin n\theta+b^n \cos n\theta$ in the form $p^2(a^{2n}+b^{2n})=r^{2n+2}$. **(7 Marks)**

(c) If $u=\log(x^3+y^3+z^3-3xyz)$ show that

$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$ and hence show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=\frac{-9}{(x+y+z)^2}$$ **(7 Marks)**

4. (a) State and prove Euler's theorem for a homogeneous function $u(x,y)$ of degree n and hence show that

$$x^2\,u_{xx}+2xy\,u_{xy}+y^2\,u_{yy}=n(n-1)u$$ **(7 Marks)**

(b) If $x = a^u \cos v$ and $y = a^u \sin v$ show that $JJ' = 1$. **(7 Marks)**

(c) The current measured by a tangent golvanometer is given by the relation $c = k \tan\theta$ where θ is the angle of deflection. Show that the relative error in c due to a given error in θ is minimum when $\theta = 45^0$. **(6 Marks)**

PART - C

5. (a) Obtain the reduction formula for $I_n = \int_0^{\frac{\pi}{4}} Sec^n x$ where n is a positive integer and hence find I_6. **(6 Marks)**

(b) Show that when n is a positive integer

$$\int_0^{2a} x^n\sqrt{2ax - x^2}\,dx = \pi a^2 \left(\frac{a}{2}\right)^n \cdot \frac{(2n+1)!}{(n+2)!n!}$$

and hence find $\int_0^{2a} x^3 \sqrt{2ax - x^2}\,dx$. **(7 Marks)**

(c) Trace the astroid : $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. **(7 Marks)**

6. (a) Find the length of an arch of the cycloid $x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$. **(6 Marks)**

(b) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta - \sin\theta);\ y = a(1 - \cos\theta)$ about the base. **(7 Marks)**

(c) Find the volume of solid generated by the revolution of the cardiod $r = a(1 + \cos\theta)$ about the initial line. **(7 Marks)**

PART - D

7. (a) Solve

i) $\frac{dy}{dx} = x\tan(y - x) + 1$

ii) $(x - 4y - 9)\,dx + (4x + y - 2)\,dy = 0$

iii) $[xy\sin(xy) + \cos(xy)]y\,dx + [xy\sin(xy) - \cos(xy)]\,xdy = 0$

(5×3=15 Marks)

(b) Find the orthogonal trajectories of the family of curves $\left(r + \frac{k^2}{r}\right)\cos\theta = a$, '$a$' being the parameter. **(5 Marks)**

8. (a) Examine the nature of the following series.

i) $\frac{1}{1^2} + \frac{1+2}{1^2+2^2} + \frac{1+2+3}{1^2+2^2+3^2} + \ldots$ **(6 Marks)**

ii) $1 + \left(\frac{2}{3}\right)x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \ldots\ldots;\ x > 0$ **(7 Marks)**

iii) $1 + \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} - \frac{1}{7^2} - \frac{1}{8^2} + \ldots$ **(7 Marks)**

** * **