NEW SCHEME

MAT21

USN 1MS04CS066

Second Semester B.E Degree Examination, July/August 2005

Common to All Branches

Engineering Mathematics II

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least one question from each part.*
2. All questions carry equal marks.

PART A

1. (a) For the curve $\theta = cos^{-1}\left(\frac{r}{k}\right) - \frac{\sqrt{k^2-r^2}}{r}$ prove that $r\frac{ds}{dr} =$ const. **(6 Marks)**

(b) State Rolle's Theorem and verify the same for

$f(x) = log\frac{x^2+ab}{x(a+b)}$ in $[a, b]$ **(7 Marks)**

(c) Find the first four non zero terms in the expansion of $f(x) = \frac{x}{e^x-1}$ using Maclaurin's series. **(7 Marks)**

2. (a) Evaluate $\lim_{x \longrightarrow a}\left(2-\frac{x}{a}\right)^{tan\frac{\pi x}{2a}}$ **(6 Marks)**

(b) Expand $f(x, y) = x^2y + 3y - 2$ as a polynomial of powers of $(x-1)$ and $(y+2)$ upto second degree terms using Taylor's theorem. **(7 Marks)**

(c) Find the dimensions of the rectangular box, open at the top, of the maximum capacity whose surface is 432 sq.cm. **(7 Marks)**

PART B

3. (a) Change the order of integration and evaluate

$$\int_0^3 \int_0^{\sqrt{4-y}} (x+y)dxdy$$ **(6 Marks)**

(b) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{asin\theta} \int_0^{\frac{a^2-r^2}{2}} rdrd\theta dz$ **(7 Marks)**

(c) Using Beta and Gamma functions evaluate

$$\int_0^{\frac{\pi}{2}} \sqrt{sin\theta}\, d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{sin\theta}}$$ **(7 Marks)**

4. (a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$. **(6 Marks)**

(b) If $\vec{v_1}$ and $\vec{v_2}$ be vectors joining the fixed points (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively to a variable point (x, y, z) prove that div$(\vec{v_1} \times \vec{v_2}) = 0$ **(7 Marks)**

(c) Verify Green's theorem for $\int_c [(xy+y^2)dx + x^2 dy]$ where c is bounded by $y = x$ and $y = x^2$ **(7 Marks)**

PART C

5. (a) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - log2$ **(6 Marks)**

(b) Solve $\frac{d^2y}{dx^2} + a^2 y = tan\, ax$ **(7 Marks)**

(c) Solve by method of variation of parameters

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$ **(7 Marks)**

6. (a) Solve by the method of undetermined coefficients

$$(D^2 - 2D)y = e^x sinx$$ **(6 Marks)**

(b) Solve $x^2\frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 logx$ **(7 Marks)**

(c) Solve the initial value problem $\frac{d^2y}{dx^2} + y = sin(x+a)$ satisfying the conditions $y(0) = 0 \;\; y'(0) = 0$ **(7 Marks)**

PART D

7. (a) i) Evaluate $L\{t(sin^3t - cos^3t)\}$

ii) Using Laplace transform evaluate

$$\int_0^\infty e^{-t} t sin^2 3t dt$$ **(6 Marks)**

(b) Find the Laplace transform of

$$f(t) = Esin\omega t \quad 0 < t < \frac{\pi}{\omega} \text{ given } f(t + \frac{\pi}{\omega}) = f(t)$$ **(7 Marks)**

(c) Express the following function in terms of Heaviside's unit step function and hence find its Laplace transform

$$f(t) = t^2 \qquad 0 < t < 2$$
$$= 4t \qquad t > 2$$ **(7 Marks)**

8. (a) Evaluate

i) $L^{-1}\left\{\frac{1}{s+3} + \frac{s+3}{s^2+6s+13} - \frac{1}{(s-2)^3}\right\}$

ii) $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$ **(6 Marks)**

(b) Evaluate

i) $L^{-1}\{cot^{-1}s\}$ ii) $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ **(8 Marks)**

(c) Using Laplace transform method solve

$$\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = t^2e^t \text{ given } y(0) = 1, y'(0) = 0, y''(0) = -2$$ **(6 Marks)**

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