First Semester B.E. Examination
Engineering Mathematics- I (06MAT11)

Time:03 Hours **Max Marks:100**

Model Question Paper

Note: 1. Answer any FIVE full question selecting at least TWO questions from each Part
2. Answer all objective types questions only in first and second writing pages.
3. Objective types questions should not be repeated.

PART - A

Q1. a (i) If $y = x^n \log x$, then y_{n+1} is

(A) (n-1)! / x (B) n! / x (C) n! / x^n (D) -n! / x^n

(ii) The angle between radius vector & tangent is

(A) $\tan\phi = r\frac{d\theta}{dr}$ (B) $\tan\phi = r\frac{dr}{d\theta}$ (C) $\tan\phi = \frac{1}{r}\frac{dr}{d\theta}$ (D) $\tan\phi = \frac{dr}{d\theta}$

(iii) The nth derivative of log (ax + b) is

(A) $\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$ (B) $\frac{(-1)^n n!a^n}{(ax+b)^{n+1}}$ (C) $\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^{n+1}}$ (D) $\frac{(-1)^{n-2}(n-2)!a^n}{(ax+b)^{n+1}}$

(iv) Angle between the two curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$ is

(A) $-3\pi/4$ (B) $\pi/4$ (C) $-\pi/4$ (D) $\pi/2$ (1x 4 = 4)

b. Find the nth derivative of $e^{-2x}\cos^3 2x$ (4)

c. If y = a cos (logx) + b sin(logx) prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ (6)

d. Find the pedal equation of the curve $r^n = a^n \cos n\theta$ (6)

Q.2. a (i) If $u = x^2 + y^2$, then $\frac{\partial^2 u}{\partial x \partial y}$ is equal to

(A) -2 (B) 2 (C) 0 (D) 2 x + 2y

(ii) If $u = f(x+ay) + g(x-ay)$,then the value $\frac{\partial^2 u}{\partial^2 y}$ is

(A) $\frac{\partial^2 u}{\partial x^2}$ (B) $a\frac{\partial^2 u}{\partial x^2}$ (C) $a^2\frac{\partial^2 u}{\partial x^2}$ (D) $\frac{\partial^2 u}{\partial x \partial y}$

(iii) If $u = \sin^{-1}\left(\frac{x}{y}\right) + ta\,n^{-1}\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

(A) -1 (B) 1 (C) 0 (D) -2

(iv) If u = x (1-y) and v = xy, then the value of JJ^1

(A) 2 (B) -1 (C) 1 (D) 0 (1x 4 = 4)

b. If u is a homogeneous function of x & y with degree n, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\,u$ (4)

c. If $u = ta\,n^{-1}\left(\frac{y}{x}\right) + y\sin^{-1}\left(\frac{x}{y}\right)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$ (6)

d. If $u = \frac{xy}{z}$, $v = \frac{yz}{x}$, $w = \frac{xz}{y}$ then verify that $JJ^1 = 1$ (6)

Q3.a. (i) The value of $\int_0^1 x^2(1-x^2)^{3/2}\,dx$ is

(A) $\pi/32$ (B) $-\pi/32$ (C) 0 (D) 1/32

(ii) The equation of the asymptote of $x^3 + y^3 = 3axy$ is

(A) x + y -a=0 (B) x - y + a =0 (C) x + y + a = 0 (D) x – y – a = 0

(iii) A curve r = a (1+cosθ) has a maximum value

(A) a (B) 2a (C) -2a (D) 0

(iv) If $In = \int \tan^n \theta\, d\theta$,then which of the following is true

(A) $n(I_{n+1} + I_{n-1}) = \mathbf{1}$ (B) $I_{n+1} + I_{n-1} = \mathbf{1}$ (C) $n(I_{n+1} - I_{n-1}) = \mathbf{1}$ (D) $I_{n+1} + I_n = \mathbf{1}$

(1x 4 = 4)

b. 0btain the reduction formula for $\int \cos^n x\, dx$ (4)

c. Evaluate $\int_0^{2a} \frac{x^2}{\sqrt{2ax - x^2}}\,dx$ (6)

d. Trace the curve $r^2 = a^2 \cos 2\theta$ (6)

Q.4 a. (i) The complete area of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ is

(A) 2a (B) -a (C) 0 (D) 6a

(ii) The length of the loop of the curve $3ay^2=x(x-a)^2$ is

(A) $2a/\sqrt{3}$ (B) $4a/\sqrt{3}$ (C) $\sqrt{3}/a$ (D) $-\sqrt{3}/4a$

(iii) The Volume of the curve r = a(1+ cosθ) about the initial line is

(A) $\frac{4\pi a^3}{3}$ (B) $\frac{2\pi a^3}{3}$ (C) $\frac{8\pi a^3}{3}$ (D) $\frac{\pi a^3}{3}$

(vi) The surface area of the sphere of radius 'a' is

(A) $2\pi r^2$ (B) $4\pi a^2$ (C) $4\pi r$ (D) $4\pi a$ (1x 4 = 4)

b. Find the length of the arc of the curve y =log secx between the points x = 0 to x= $\pi/3$ (4)

c. Find the area bounded by the curve $r^2=a^2\cos 2\theta$ (6)

d. Evaluate $\int_0^{\pi}\log(1+\alpha\cos x)dx$, using differentiation under integral sign (6)

PART -B

Q5 a. (i) The solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\tan\left(\frac{y}{x}\right)$ is

(A) cos(x/y) = c (B) sin(y/x) = c (C) $\sin^{-1}(x/y)=cx$ (D) cos(y/x)=cx

(ii) The solution of the differential equation $\frac{dy}{dx}=\frac{y}{x+\sqrt{xy}}$ is

(A) $2\sqrt{\frac{y}{x}}+\log y=c$ (B) $2\sqrt{\frac{x}{y}}=\log y+c$ (C) $-3\sqrt{\frac{x}{y}}+\log x=c$ (D) $\log\frac{x}{y}-\frac{x}{y}=c$

(iii) The integrating factor of the differential equation $\frac{dy}{dx}+y\cot x=\cos x$ is

(A) cosx (B) - sinx (C) sinx (D) cotx

(iv) The Orthogonal trajectory of xy = c is

(A) $x^2-y^2=c$ (B) $x^2+y^2=c$ (C) $x-y=c^2$ (D) $x^2-y=c$ (1x 4 = 4)

b Solve $3\, e^x \tan y dx + (1-e^x) \sec^2 y\, dy = 0$ (4)

c Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$ (6)

d Find the Orthogonal trajectory of the curve $y^2 = cx^3$,where c is the parameter . (6)

Q6 a (i) $\sum\left(1+\frac{1}{n}\right)^{-n^2}$ is

(A)Convergent (B) Oscillatory (C) Divergent (D) Conditionally convergent

(ii)If $\lim_{n\to\infty}(u_n)^{1/n} = l$ then $\sum u_n$ is convergent for

(A) $l<1$ (B) $l>1$ (C) $l \le 1$ (D) $l>-1$

(iii) A sequence which is monotonic and bounded is

(A)Absolutely Convergent (B) Oscillatory (C) Divergent (D) Convergent

(iv) The series $1-\frac{1}{5}+\frac{1}{9}-\frac{1}{13}+--------$ is

(A)Absolutely Convergent (B) Oscillatory

(C) Conditionally convergent (D) convergence (1x 4 = 4)

b. Determine the nature of the series $\sum\left(\frac{2n^3+5}{4n^2+1}\right)$ (4)

c Show that the series $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ is convergent (6)

d. Test the series $\frac{x}{\sqrt{3}}-\frac{x^2}{\sqrt{5}}+\frac{x^3}{\sqrt{7}}---------$ for

(i)Absolute Convergence (ii) Conditional convergence. (6)

Q7. a (i) Three lines are coplanar if

(A) they are concurrent (B) a line is parallel

(C) a line is perpendicular to each of them

(D) they are concurrent and a line is perpendicular to each of them

(ii) The angle between two diagonals of a cube is

(A) $\theta = \cos^{-1}(2/3)$ (B) $\theta = \cos^{-1}(1/3)$ (C) $\theta = \cos^{-1}(4/3)$ (D) $\theta = \tan^{-1}(1/3)$

(iii) The general equation of the plane is

(A) ax + by =d (B) ax + by- cz = d (C) ax + by + cz + d=0 (D) ax-by = 0

(iv) The angle between the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the plane $x+2y+3z+4=0$ is

(A) $\theta = \cos^{-1}\left(3/5\right)$ (B) $\theta = \sin^{-1}\left(2/7\right)$ (C) $\theta = \cos^{-1}\left(-1/2\right)$ (D) $\theta = -\sin^{-1}\left(2/7\right)$ (1x 4 = 4)

b Find the direction ratios and the direction cosines of the line segment joining the points P(1,2,-3) and Q(3,0,-4) (4)

c Find the image of the point (1,2,3) in the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z}{-1}$ (6)

d . Find the angle between the lines $2x+2y-z+15=0=4y+z+29$ and $\frac{x+4}{4} = \frac{y-3}{-3} = \frac{z+2}{1}$ (6)

Q.8 a (i) If $\vec{r} = xi + yj + zk$, then $\nabla . \vec{r}$ is

(A) 0 (B) 1 (C) 2 (D) 3

(ii)The value of ∇r^n is

(A) $n(n-1)r^n$ (B) $n(n+1)r^{n-2}$ (C) $n(n+1)r^n$ (D) $n(n-1)r^{n-3}$

(iii) The directional derivative of $xy^2 + yz^3$ at the point (2,-1,1) in the direction of vector i+2j +2k is

(A) 11/3 (B) 10/3 (C) -11/3 (D) 3/11

(iv) Any motion in which the curl of the velocity vector is zero is said to be

(A) Rotational (B) Scalar (C) Field (D) Irrotational (1x 4 = 4)

b Find the unit tangent vector to the curve $r = t^2i + 2tj - t^3k$ at the points $t = \pm 1$ (4)

c If $\phi = x^3+y^3+z^3-3xyz$,then find $\nabla\phi$ and $|\nabla\phi|$ at the point P(1,-1 ,2) (6)

d If f and g are irrotational vector fields.Show that f x g is a solenoidal vector (6)

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