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ENTRANCE EXAMINATIONS, JUNE 2010

QUESTION PAPER BOOKLET

Ph.D. (PHYSICS)

Marks: 75
Time: 2.00 hrs.

Hall Ticket No.:

I. Please **enter your Hall Ticket Number on Pages 1 and OMR sheet** without fail

II. Read carefully the following instructions:

This Question paper has Two Sections: **Section- A** and **Section- B**

1. **Section- A** consists of 25 objective type questions of one mark each. **There is negative marking of 0.33 mark for every wrong answer.** The marks obtained by the candidate in this part will be used for resolving tie cases.

2. **Section- B** consists of 50 objective type questions of one mark each. **There is no negative marking in this section.**

3. Answers are to be marked on the OMR answer sheet following the instructions provided there upon.

   100. (A) (B) (C) ●

4. Calculators are permitted. Logarithmic tables are not allowed.

5. All questions are to be answered.

6. Handover both question booklet and the OMR sheet at the end of examination.

This book contains 26 pages

III. Values of physical constants:
$c = 3 \times 10^{8}$ m/s ; $h = 6.63 \times 10^{-34}$ J.s ; $k_B = 1.38 \times 10^{-23}$ J/K
$e = 1.6 \times 10^{-19}$ C ; $\mu_0 = 4\pi \times 10^{-7}$ Henry/m ; $\varepsilon_0 = 8.85 \times 10^{-12}$ farad/m

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SECTION - A

1. Let

$$f(x) = \begin{cases} \frac{1}{|x|}, & x \geq 1 \\ ax^2 + b, & |x| < 1 \end{cases}$$

For the function $f(x)$ to be continuous and differentiable at any point, a and b should have the values

A. $a = 1, b = 0$

B. $a = 1/2, b = 1/2$

C. $a = 1/2, b = 3/2$

D. $a = -1/2, b = 3/2$

2. The sum of the squares of the eigenvalues of a 3×3 matrix A is 20. The corresponding sum for the matrix $2A$ is

A. 40

B. 60

C. 120

D. 80

3. The differential equation and its general solution for the roots of the characteristics equations $\lambda_1 = 0,\ \lambda_2 = \lambda_3 = 4$ are given by

A. $y''' - 8y'' + 16y' = 0;\ y = c_1 + (c_2 + c_3 x)\cosh 4x$

B. $y''' - 8y'' + 16y' = 0;\ y = c_1 + (c_2 + c_3 x)e^{4x}$

C. $y''' - 8y'' + 16y' = 0;\ y = c_1 + (c_2 + c_3 x)\sin 4x$

D. $y''' - 8y'' + 16y' = 0;\ y = c_1 + (c_2 + c_3 x)e^{4x}$

4. If θ is real, the function $\sin(\theta^2)$ is

A. a periodic function of θ with period 2π

B. a periodic function of θ with period $(2\pi)^2$

C. a periodic function of θ with period π^2

D. not a periodic function of θ

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5. One mole of a monatomic ideal gas initially at temperature T_o expands from volume V_o to $2V_o$ at constant pressure. The work of expansion (W) and the heat absorbed by the gas (Q) are:

A. $W = RT_o, \;\; Q = \frac{5}{2}RT_o$

B. $W = RT_o \ln 2, \;\; Q = RT_o$

C. $W = 3RT_o, \;\; Q = \frac{3}{2}RT_o$

D. $W = \frac{3}{2}RT_o, \;\; Q = \frac{5}{2}RT_o$

where R is the gas constant.

6. A body of constant heat capacity C_p and a temperature T_i is brought into contact with a reservoir at temperature T_f. If the equilibrium between the body and the reservoir is established at constant pressure and $T_i \neq T_f$, the total entropy change of the body plus the reservoir is

A. $C_p \ln(T_f/T_i)$

B. $C_p(T_i - T_f)/T_f$

C. $C_p[\ln(T_f/T_i) - 1 + (T_i/T_f)]$

D. $C_p[(T_i/T_f) - \ln(T_f/T_i)]$

7. If Z is the canonical partition function of a system, the entropy of the system is

A. $k_B \ln Z$

B. $k_B T(\partial \ln Z/\partial V)_T$

C. $k_B T \ln Z$

D. $k_B[\ln Z + T(\partial \ln Z/\partial T)_P]$

8. If the interatomic potential energy in a solid is given by $U(r) = \frac{b}{r^n} - \frac{a}{r^m}$ where a and b are positive constants and $n > m$, the equilibrium interatomic spacing, r_o, is

A. $r_o = \left[\left(\frac{a}{b}\right)\left(\frac{m}{n}\right)\right]^{1/(m+n)}$

B. $r_o = \left[\left(\frac{b}{a}\right)\left(\frac{n}{m}\right)\right]^{1/(n-m)}$

C. $r_o = \left[\left(\frac{a}{b}\right)\left(\frac{n}{m}\right)\right]^{1/(n-m)}$

D. $r_o = \left[\left(\frac{b}{a}\right)\left(\frac{m}{n}\right)\right]^{1/(m+n)}$

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9. For non-magnetic metals, specific heat (C) at low temperatures ($T \ll \Theta_D$, the Debye temperature) varies with temperature (T) as

A. $\frac{C}{T} = aT$
B. $\frac{C}{T} = a + bT$
C. $\frac{C}{T} = a$
D. $\frac{C}{T} = a + bT^2$

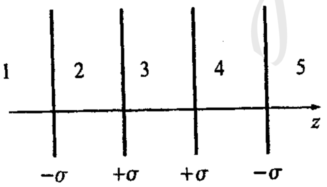
10. A particle of mass m whose total energy is twice its rest energy collides with an identical particle at rest. If they stick together, the mass M and the velocity v of the resulting composite particle are given by

A. $M = \sqrt{6}\,m,\ v = c/\sqrt{3}$
B. $M = \sqrt{3}\,m,\ v = c\sqrt{2/3}$
C. $M = 2m,\ v = c\sqrt{5/3}$
D. $M = \sqrt{2}\,m,\ v = c\sqrt{7/3}$

11. In a Compton scattering experiment a photon of energy E_0 is scattered by an electron (of rest mass m) at rest. The energies of the photons scattered at 0°, 90° and 180° with respect to the incident photon are in the proportion

A. $1 : \dfrac{mc^2}{2E_0 + mc^2} : \dfrac{mc^2}{E_0 + mc^2}$
B. $1 : \dfrac{mc^2}{E_0 + mc^2} : \dfrac{mc^2}{2E_0 + mc^2}$
C. $1 : \dfrac{mc^2}{2E_0 + mc^2} : \dfrac{mc^2}{E_0 - mc^2}$
D. $1 : \dfrac{mc^2}{E_0 + mc^2} : \dfrac{\sqrt{2}mc^2}{E_0(\sqrt{2} - 1) + \sqrt{2}mc^2}$

12. Consider 4 infinite sheets of surface charge density as shown in the figure.



The field in region 2 is given by

A. 0
B. $\sigma/\epsilon_0 \hat{z}$
C. -$\sigma/(2\epsilon_0)\hat{z}$
D. -$\sigma/\epsilon_0 \hat{z}$

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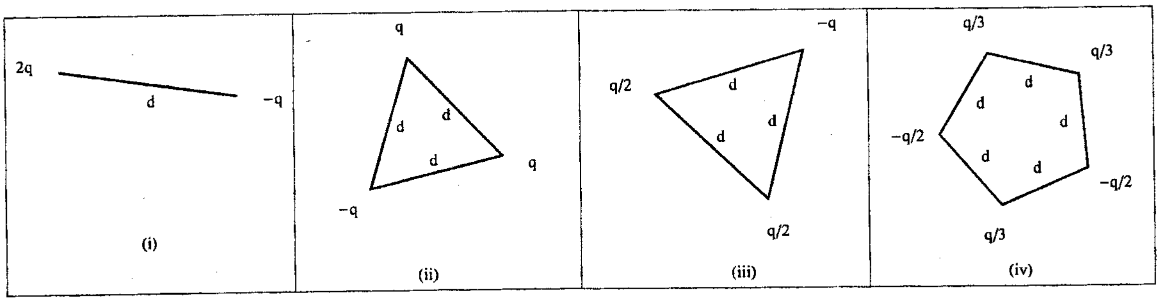
13. Consider the following charge distributions. All the polygons are regular with edge length d.



The dipole moment is independent of the choice of origin for configurations

A. (ii) and (iii)
B. (i) and (iv)
C. (iii) and (iv)
D. (i) and (ii)

14. A hollow metallic waveguide can support

A. TE, TM and TEM waves
B. Either TE or TM waves
C. Both TE and TM waves
D. only TEM waves

15. To use a transistor as an amplifier

A. emitter-base junction should be forward-biased and the collector-base junction reverse-biased
B. both junctions should be forward-biased
C. both junctions should be reverse-biased
D. it does not matter how the transistor is biased, it always works as an amplifier

16. Which of the following logic gates can be used as a controlled inverter?

A. AND gate
B. OR gate
C. INVERTER gate
D. XOR gate

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17. The neutrinos emitted in a β decay process are

A. right handed

B. left handed

C. both left and right handed

D. spin-zero and massless particles

18. In the fission of $^{235}U_{92}$, the mass ratio of the two fission fragments is 1.5. What is the ratio of the velocities of these fragments?

A. 3 : 2

B. 2 : 3

C. 9 : 4

D. 4 : 9

19. Alpha decay is an example of

A. nuclear fission

B. Barrier penetration

C. Coulomb repulsion

D. nuclear ionization

20. Let $|n>$ denote the normalised eigenstates of the Hamiltonian H describing a linear harmonic oscillator corresponding to the eigenvalues $(n+\frac{1}{2})\hbar\omega$. The normalised state

$$|\psi>= \frac{1}{\sqrt{5}}[2|1> -|2>]$$

is

A. an eigenstate of H corresponding to the eigenvalue $\frac{1}{2}\hbar\omega$

B. an eigenstate of H corresponding to the eigenvalue $\frac{11}{2}\hbar\omega$.

C. an eigenstate of H corresponding to the eigenvalue 0

D. not an eigenstate of H.

21. $\hat{A}$, $\hat{B}$ and $\hat{C}$ are three Hermitian operators. Which of the following statements *cannot* be correct?

A. $[\hat{A},\hat{B}] = \hat{C}$

B. $\{A, B\} = \hat{C}$

C. $[A, B] = i\hat{C}$

D. $\hat{A}^2 + \hat{B}^2 = \hat{C}^2$

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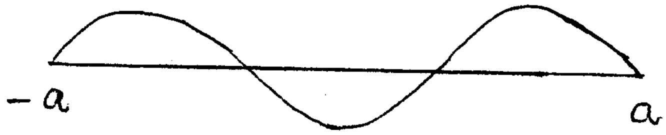
22. The allowed energies for a particle in a one-dimensional square well potential between $-a$ and a are

$$E_n = \frac{\pi^2\hbar^2 n^2}{8ma^2}$$

The wave function shown in the figure



corresponds to

A. $n - 3$

B. $n = 2$

C. $n = 4$

D. $n = 7$

23. With the positive z-axis chosen to be upwards, the Lagrangian of a particle of mass m falling under gravity is

A. $\frac{1}{2}m\dot{z}^2 - mgz$

B. $-\frac{1}{2}m\dot{z}^2 - mgz$

C. $\frac{1}{2}m\dot{z}^2 + mgz$

D. $-\frac{1}{2}m\dot{z}^2 + mgz$

24. An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900 km/h. The ratio of horizontal acceleration of the aircraft to the acceleration due to gravity (g=9.8 m/sec^2) is about

A. 0

B. 5

C. 6

D. 7

25. Which of the following has non-vanishing Poisson bracket with L_Z, the Z-component of the angular momentum $\vec{L}$?

A. $a(x^2 + y^2 + z^2)$

B. $a(x^2 + y^2) + bz^2 \quad a \neq b$

C. $ax^2 + by^2 + az^2 \quad a \neq b$

D. $a(x^2 + y^2)^2$

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SECTION - B

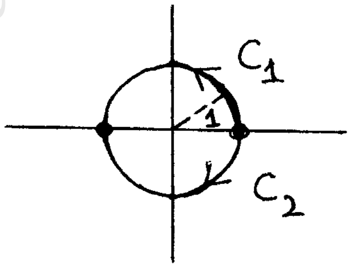
26. The value of the integral

$$\int_{-1}^{2} dx\, x^2 \delta(2x-1)$$

where $\delta(x)$ denotes the Dirac delta function, is

A. 0
B. 1/4
C. 1
D. 1/8

27. If I_1 and I_2 denote the values of the integral of $\frac{1}{(Z+\frac{1}{2})^2}$ evaluated along the semicircle C_1 and C_2 as shown, then $I_1 - I_2$ is



A. 0
B. $2\pi i \times \frac{1}{4}$
C. $2\pi i \times \frac{1}{8}$
D. 2π

28. For a matrix A satisfying $A^3 = \mathbf{1}$, which of the following *cannot* be a possible set of eigenvalues?

A. $(1, 1, 1)$
B. $(1, 1, e^{2\pi i/3})$
C. $(1, e^{2\pi i/3}, e^{4\pi i/3})$
D. $(1, -e^{2/\pi i/3}, e^{4\pi i/3})$

29. It is impossible to find a real 4×4 matrix with the eigenvalues given by

A. $(1, 1, -1, -1)$
B. $(1, i, i, 1)$
C. $(1, i, -i, 1)$
D. $(1, 1, 0, 0)$

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30. The value of the integral

$$\int_{-\infty}^{\infty} e^{-x^2} H_3(x) P_2(x) dx$$

where $H_n(x)$ and $P_n(x)$ are the Hermite and the Legendre polynomials, is

A. $2^3 \cdot 3! \sqrt{\pi}$

B. $2/5$

C. $\sqrt{2^3 \cdot 3! \sqrt{\pi} \cdot \frac{2}{5}}$

D. 0

31. For the function $f(z) = \dfrac{z}{(z+2)^2}$ the order of the pole at $z = -2$ and the residue at the pole are respectively given by

A. 2, -2

B. 2, 1

C. 1, -2

D. 1, 1

32. The Bernoulli equation given by $y' = P(x)y + Q(x)y^m, m \neq 0, m \neq 1$, can be reduced to a linear equation for z by substitution

A. $z = y^{1-m}$

B. $z = y^{m-1}$

C. $z = y^{1-2m}$

D. $z = y^{2m-1}$

33. The rank of the matrix

$$\begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

is

A. 1

B. 2

C. 3

D. 4

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34. Let $|n>$ denote the eigenstates of a linear harmonic oscillator corresponding to the eigenvalues $(n+\frac{1}{2})\hbar\omega$. If $|\psi(0)>=\frac{1}{\sqrt{2}}[|0>+|1>]$ denotes the state of a harmonic oscillator at $t=0$ and $|\psi(t)>$ that at time t, the $|<\psi(0)|\psi(t)>|^2$ first becomes zero at

A. $t=\pi/8\omega$

B. $t=\pi/4\omega$

C. $t=\pi/2\omega$

D. $t=\pi/\omega$

35. Which of the following pairs of wave function (with $A\neq B$) describe the same physical state of a free particle

A. $\psi_1(x)=Ae^{ikx}+Be^{-ikx}$, $\psi_2(x)=-Ae^{ikx}+Be^{-ikx}$

B. $\psi_1(x)=Ae^{ikx}+Be^{-ikx}$, $\psi_2(x)=Ae^{ikx}-Be^{-ikx}$

C. $\psi_1(x)=Ae^{ikx}+Be^{-ikx}$, $\psi_2(x)=-Ae^{ikx}-Be^{-ikx}$

D. $\psi_1(x)=Ae^{ikx}+Be^{-ikx}$, $\psi_2(x)=Be^{ikx}+Ae^{-ikx}$

36. For a spin- $\frac{1}{2}$ particle in the state $\begin{pmatrix}\alpha_+\\ \alpha_-\end{pmatrix}$, the expectation value of $\hat{S}_z=\frac{1}{2}\hbar\sigma_z$ is

A. $\frac{\hbar}{2}(\alpha_+^*\alpha_-+\alpha_-^*\alpha_+)$

B. $\frac{-i\hbar}{2}(\alpha_+^*\alpha_--\alpha_-^*\alpha_+)$

C. $\frac{\hbar}{2}(|\alpha_+|^2-|\alpha_-|^2)$

D. 0

37. Two spin- $\frac{1}{2}$ particles with spins $\hat{S}_1$ and $\hat{S}_2$ are in a singlet state. The value of the potential $V(r)=V_1(r)+\frac{1}{\hbar^2}\hat{S}_1\cdot\hat{S}_2V_2(r)$, for such a state will be

A. $V_1(r)+\frac{1}{4}V_2(r)$

B. $V_1(r)-\frac{3}{4}V_2(r)$

C. 0

D. $V_1(r)-\frac{3}{2}V_2(r)$

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38. If $\hat{H}$ and $\hat{p}$ respectively denote the Hamiltonian and the momentum operator for a free particle, then the functions $\sin kx$ and $\cos kx$ are eigenfunctions

A. of both $\hat{H}$ and $\hat{p}$

B. only of $\hat{H}$ but not of $\hat{p}$

C. only of $\hat{p}$ but not of $\hat{H}$

D. of neither $\hat{H}$ nor $\hat{p}$

39. If the Gibbs free energy of a system is given by $G(T,P) = RT\ln[\alpha P/(RT)^{5/2}]$, where α and R are constants, the specific heat at constant pressure, C_p, is

A. $\frac{3}{2}R$

B. $\frac{5}{2}R$

C. $\frac{5}{2}R - R\ln[\alpha P/(RT)^{5/2}]$

D. $R\ln[\alpha P/(RT)^{5/2}]$

40. The entropy of a magnetic system changes by an amount $\Delta S = -CH\Delta H/T^2$ (where C is a constant) if its temperature is held constant at T and the magnetic field is changed from H to $H+\Delta H$. The magnetization M of such a system depends on temperature T and field H as

A. $M = CH^2/T$

B. $M = CT/H$

C. $M = C(T/H)^2$

D. $M = CH/T$

Hint: the change in the Gibbs free energy is given by $dG = -SdT - MdH$

41. A system of two energy levels ϵ_o and ϵ_1 ($\epsilon_1 > \epsilon_o$) is populated by N particles at temperature T in accordance with the Boltzmann distribution law. The average energy per particle is

A. $\epsilon_o + \epsilon_1 e^{-(\epsilon_1-\epsilon_o)/k_BT}$

B. $\epsilon_1 \pm \epsilon_o e^{-(\epsilon_1-\epsilon_o)/k_BT}$

C. $[\epsilon_o + \epsilon_1 e^{-(\epsilon_1-\epsilon_o)/k_BT}]/[1 + e^{-(\epsilon_1-\epsilon_o)/k_BT}]$

D. $[\epsilon_o + \epsilon_1 e^{-(\epsilon_1-\epsilon_o)/k_BT}]/[1 - e^{-(\epsilon_1-\epsilon_o)/k_BT}]$

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42. A monatomic gas consists of atoms with two internal energy levels: a ground state of degeneracy g_1 and an excited state of degeneracy g_2 at an energy ϵ above the ground state (chosen as the zero of energy scale). The specific heat of the gas is

A. $\frac{3}{2}k_B$

B. $\frac{3}{2}k_B - \frac{g_1 e^{-\epsilon/k_BT}}{1 + g_2 e^{-\epsilon/k_BT}}$

C. $\frac{3}{2}k_B + \frac{g_1 g_2 \epsilon^2 e^{\epsilon/k_BT}}{k_BT^2(g_2 + g_1 e^{\epsilon/k_BT})^2}$

D. $\frac{g_1 g_2 \epsilon^2 e^{-\epsilon/k_BT}}{k_BT(g_2 + g_1 e^{\epsilon/k_BT})^2}$

43. Consider an ideal gas consisting of N particles obeying classical statistics. Suppose that the energy of one particle ϵ is proportional to the magnitude of its momentum p, i.e., $\epsilon = Cp$. The partition function of the gas is proportional to

A. $\left(\frac{k_BT}{C}\right)^{3N}$

B. $\left(\frac{k_BT}{C}\right)^{N}$

C. $\left(\frac{C}{k_BT}\right)^{N}$

D. $(Ck_BT)^N$

44. If in a diffraction experiment, X-ray photon of wavevector $\vec{k}$ gets scattered to the final state with wave vector $\vec{k'} = \vec{k} + \vec{G}$, where $\vec{G}$ is a reciprocal lattice vector, the Bragg diffraction condition is given by

A. $\vec{k} \cdot \vec{G} + \vec{G} = 0$

B. $\vec{k} \cdot \vec{G} + G^2 = 0$

C. $\vec{k} \cdot \vec{G} = (\vec{G} \cdot \vec{G})/2$

D. $\vec{k} \cdot \vec{k} = 2(\vec{k} \cdot \vec{G})$

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45. If $\vec{a}_1 = \frac{a}{2}(-\hat{x} + \hat{y} + \hat{z})$; $\vec{a}_2 = \frac{a}{2}(\hat{x} - \hat{y} + \hat{z})$ and $\vec{a}_3 = \frac{a}{2}(\hat{x} + \hat{y} - \hat{z})$ are the primitive translation vectors of a cubic lattice of size (lattice parameter) a and $\hat{x}, \hat{y}, \hat{z}$ are the orthogonal unit vectors parallel to the cube edges, the primitive translation vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$ of the reciprocal lattice are given by

A. $\vec{b}_1 = \frac{2\pi}{a}(\hat{y} + \hat{z})$; $\vec{b}_2 = \frac{2\pi}{a}(\hat{z} + \hat{x})$; $\vec{b}_3 = \frac{2\pi}{a}(\hat{x} + \hat{y})$;

B. $\vec{b}_1 = \frac{2\pi}{a}(\hat{y} - \hat{z})$; $\vec{b}_2 = \frac{2\pi}{a}(\hat{z} - \hat{x})$; $\vec{b}_3 = \frac{2\pi}{a}(\hat{x} - \hat{y})$;

C. $\vec{b}_1 = \frac{2\pi}{a}(\hat{y} + \hat{z})$; $\vec{b}_2 = \frac{2\pi}{a}(-\hat{x} + \hat{z})$; $\vec{b}_3 = \frac{2\pi}{a}(-\hat{x} + \hat{y})$;

D. $\vec{b}_1 = \frac{2\pi}{a}(-\hat{y} + \hat{z})$; $\vec{b}_2 = \frac{2\pi}{a}(-\hat{z} + \hat{x})$; $\vec{b}_3 = \frac{2\pi}{a}(-\hat{x} + \hat{y})$;

46. If the atoms vibrating in a lattice at finite temperature are considered to be a collection of N identical harmonic oscillators, as Einstein did, the heat capacity at constant volume for such a solid is given by

A. $C_V = Nk_B x \frac{e^x}{e^x - 1}$

B. $C_V = Nk_B x^2 \frac{e^x}{e^x - 1}$

C. $C_V = Nk_B x^2 \frac{e^x}{(e^x - 1)^2}$

D. $C_V = Nk_B x \frac{e^x}{(e^x - 1)^2}$

where $x = \hbar\omega/k_B T$

47. Consider a system of N weakly interacting particles, each of spin $\frac{1}{2}$ and magnetic moment μ_B, exposed to an external magnetic field H. If the system is in thermal contact with a heat reservoir at temperature T, the mean energy of the system as a function of T and H is given by

A. $N\mu_B \sinh(\mu_B H/k_B T)$

B. $N\mu_B \coth(\mu_B H/k_B T)$

C. $N\mu_B \tanh^{-1}(\mu_B H/k_B T)$

D. $N\mu_B \tanh(\mu_B H/k_B T)$

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48. Which of the following relations is consistent with the London equation $\text{curl}\vec{j} = -\frac{c}{4\pi\lambda_L^2}\vec{B}$ for a superconductor, where λ_L is the London penetration depth.

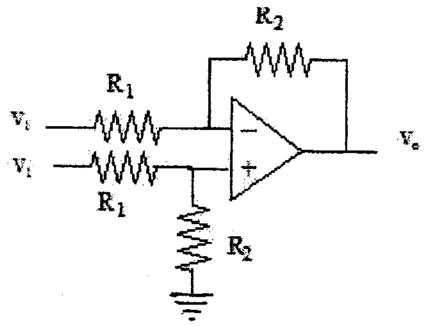
A. $\nabla\vec{B} = \vec{B}/\lambda_L$

B. $\nabla^2\vec{B} = \vec{B}/\lambda_L$

C. $\nabla^2\vec{B} = \vec{B}/\lambda_L^2$

D. $\nabla\vec{B} = \vec{B}/\lambda_L^2$

49. The output V_o of the circuit is



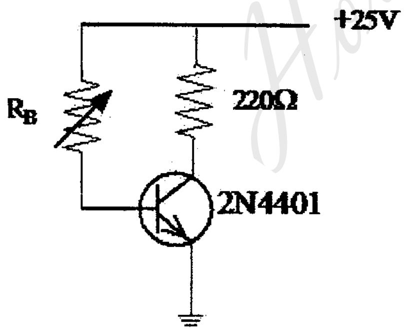
A. $V_o = 0$

B. $V_o = (R_2/R_1)V_1$

C. $V_o = (1 + R_2/R_1)V_1$

D. $V_o = -(R_2/R_1)V_1$

50. In the following figure, the transistor has β_{dc} of 80 and $V_{CE(sat)}$ of 0.1 V. R_B is adjusted to get transistor into saturation. The value of $I_{C(sat)}$ and the corresponding value of R_B are



A. 114 mA and 17 kΩ

B. 1.43 mA and 17 kΩ

C. 114 mA and 1.7 kΩ

D. 1.43 mA and 17.48 kΩ

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51. In emitter follower configuration of a transistor

A. the emitter current follows the collector current and both are in same phase

B. the collector current follows the emitter current and both are in same phase

C. the base current follows the emitter current and is in opposite phase

D. the emitter current follows the base current and both are in same phase

52. Binary equivalent of $(8.2)_{10}$ is

A. $(1\,0\,0\,1\,.\,0\,0\,1\,1)_2$

B. $(1\,0\,0\,0\,.\,0\,0\,1\,1)_2$

C. $(1\,0\,0\,0\,.\,0\,1\,1\,0)_2$

D. $(0\,1\,1\,1\,.\,0\,1\,1\,0)_2$

53. In a pure germanium the intrinsic concentration of charge (n_i) at room temperature is 2.5×10^{13} cm^3. The electron and hole mobilities are given by $\mu_n = 3800$ cm^2/V-s and $\mu_p = 1800$ cm^2/V-s respectively. The resistivity of the intrinsic germanium is

A. 22.3 Ω-cm

B. 66.9 Ω-cm

C. 44.6 Ω-cm

D. 11.2 Ω-cm

54. The change in isospin (ΔI) in the process $\Lambda^\circ \to p + \pi^-$ is

A. $\Delta I = 0, 1$

B. $\Delta I = \frac{1}{2}, \frac{3}{2}$

C. $\Delta I = 0, \frac{1}{2}$

D. $\Delta I = 1, 2$

55. A spin-0 particle A decaying in its rest frame into particles B and C, through strong interaction. Which of the following statement is not true?

A. $0^+ \to 0^+ + 0^+$

B. $0^+ \to 0^- + 0^-$

C. $0^- \to 0^+ + 0^-$

D. $0^- \to 0^- + 0^-$

where $J^p = 0^+$ or 0^- representing the standard convention for labeling the spin and intrinsic parity of a particle.

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56. Which of the following reactions represents a strong interaction

A. $\pi^- + p \to \Lambda + K^o$

B. $\rho^o \to \pi^o + \pi^o$

C. $K^- \to \pi^- + \pi^o$

D. $p + \bar{p} \to \pi^+ + \pi^o$

57. In the nuclear shell model, ${}^{16}_{8}O$ is a good closed-shell nucleus and has spin parity $J^p = 0^+$. What are the predicted J^p values for ${}^{15}_{8}O$ and ${}^{17}_{8}O$?

A. 0^+, 0^+

B. $\frac{1}{2}^-$, $\frac{5}{2}^+$

C. $\frac{3}{2}^-$, $\frac{5}{2}^+$

D. 0^+, $\frac{1}{2}^+$

58. A pion at rest decays to a muon and a neutrino. The magnitude of momentum of the outgoing muon is

A. $\dfrac{m_\pi^2 - m_\mu^2}{2m_\pi} \cdot c$

B. $\dfrac{m_\pi^2 + m_\mu^2}{2m_\pi} \cdot c$

C. $\dfrac{m_\pi^2 + m_\mu^2}{2m_\mu} \cdot c$

D. $\dfrac{(m_\pi - m_\mu)^2}{2m_\pi} \cdot c$

59. An experiment is performed to search for the evidence of the particle H through the strong reaction $p+p \to H + K^+ + K^+$. The electric charge (Q), Baryon number (B) and strangeness quantum number (S) of the particle H will be

A. $Q = 0$, $B = 0$, $S = -2$

B. $Q = 0$, $B = 2$, $S = -2$

C. $Q = 0$, $B = 2$, $S = 2$

D. $Q = 0$, $B = 2$, $S = 0$

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60. Let a plane monochromatic wave be totally internally reflected at the interface $z = 0$ between two dielectrics. Let $x - z$ be the plane of incidence with x along the interface. The planes of constant amplitudes and constant phases in medium 2 are given by

A. $z = c_1, \ x = c_2$

B. $x + z = c_1, \ x - z = c_2$

C. $x = c_1, \ z = c_2$

D. $x - z = c_1, \ x + z = c_2$

(here c_1 and c_2 are constants)

61. The transformation that reduces the wave equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2} = 0$$

to the form $\dfrac{\partial^2 u(\xi,\tau)}{\partial\xi\partial\tau} = 0$ is given by

A. $\xi = c/(ct + x), \ \tau = c/(ct - x)$

B. $\xi = it + x/c, \ \tau = it - x/c$ where $i = \sqrt{-1}$

C. $\xi = t + x/c, \ \tau = t - x/c$

D. $\xi = t + ix/c, \ \tau = t - ix/c$ where $i = \sqrt{-1}$

62. In the domain of anomalous dispersion, the group velocity (v_g), refractive index (n) and speed of light in vacuum (c) obey

A. $v_g > c/n$

B. $v_g = c/n$

C. $v_g < c/n$

D. $c/n < v_g < c$

63. The resolving power of a grating of a given width for a given order of diffraction can be increased by

A. making thinner grooves without changing the groove separation

B. making thinner grooves and proportionately decreasing the groove separation

C. making wider grooves without changing the groove separation

D. making wider grooves and proportionately increasing the groove separation

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64. In a Newton's ring experiment, a plano-convex lens with radius of curvature of the convex surface 4.18m is placed on an optically flat surface with its convex surface down. Interference fringes are observed using a travelling microscope and sodium light ($\lambda = 589.3\ nm$). The radius of the tenth dark ring is

A. 4.96 mm

B. 6.92 mm

C. 1.63 mm

D. 2.45 mm

65. Consider a uniformly charged sphere of radius R with total charge q. Let the sphere be coated with a medium with charge density $\rho = \alpha/r$ (where $\alpha > 0$, $r \geq R$). The electric field $\vec{E}$ outside the charged sphere will be independent of r when

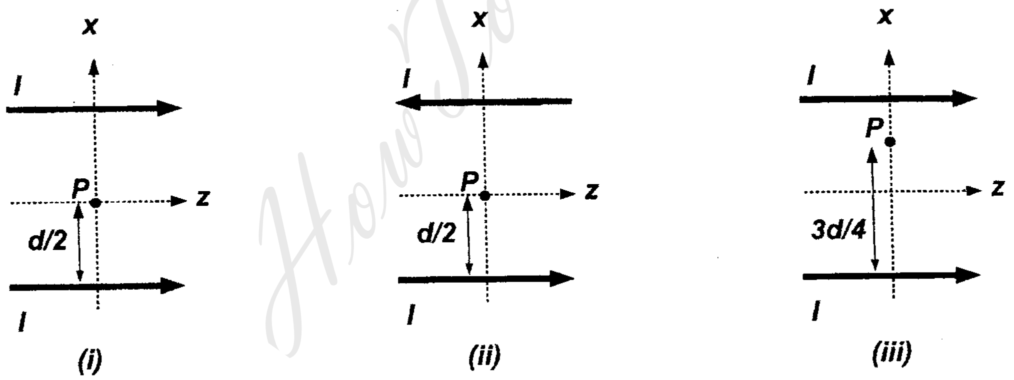
A. $q = 2\pi\alpha R^2/\epsilon_0$, $\vec{E} = \alpha/(2\epsilon_0)\ \hat{r}$

B. $q = 2\pi\alpha R^2$, $\vec{E} = \alpha/(\epsilon_0)\ \hat{r}$

C. $q = \pi\alpha R^2$, $\vec{E} = \alpha/(2\epsilon_0)\ \hat{r}$

D. $q = 2\pi\alpha R^2$, $\vec{E} = \alpha/(2\epsilon_0)\ \hat{r}$

66. Consider two infinite current carrying wires rigidly held parallel to the z-axis. The current through the wires is I. The distance between the wires is d. The magnetic field at the points marked P in the three cases shown in the figure below are



A. (i) $\frac{2\mu_0 I}{\pi d}\hat{y}$; (ii) 0; (iii) $-\frac{4\mu_0 I}{3\pi d}\hat{y}$

B. (i) 0; (ii) $-\frac{2\mu_0 I}{\pi d}\hat{y}$; (iii) $\frac{4\mu_0 I}{3\pi d}\hat{y}$

C. (i) 0; (ii) $\frac{2\mu_0 I}{\pi d}\hat{y}$; (iii) $-\frac{4\mu_0 I}{3\pi d}\hat{y}$

D. (i) $-\frac{2\mu_0 I}{\pi d}\hat{y}$; (ii) 0; (iii) $-\frac{4\mu_0 I}{3\pi d}\hat{y}$

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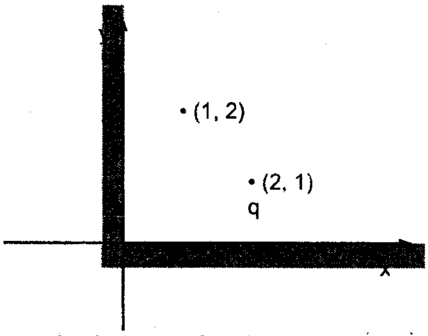
67. Two infinite conducting planes intersect at the origin (see figure). All distances are in meters.



A charge q is placed at (2,1). The potential at the point (1,2) will be

A. $\frac{q}{4\pi\epsilon_0}\frac{4\sqrt{5}-6}{3\sqrt{10}}$

B. $\frac{q}{4\pi\epsilon_0}\frac{2\sqrt{5}-6}{3\sqrt{10}}$

C. $\frac{q}{4\pi\epsilon_0}\frac{4\sqrt{5}+6}{3\sqrt{10}}$

D. $\frac{q}{4\pi\epsilon_0}\frac{-2\sqrt{5}-6}{3\sqrt{10}}$

68. The mast of a sailboat makes an angle θ with respect to its deck. An observer standing on the dock sees the boat go by with a speed v. The angle made by the mast as seen by the observer is

A. $\tan^{-1}\left(\sqrt{1-v^2/c^2}\tan\theta\right)$

B. $\tan^{-1}\left(\frac{\tan\theta}{\sqrt{1-v^2/c^2}}\right)$

C. $\tan^{-1}\left(\tan(\theta\sqrt{1-v^2/c^2})\right)$

D. $\tan^{-1}\left(\tan(\theta/\sqrt{1-v^2/c^2})\right)$

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69. The coherence matrix for light propagating along z-direction can be written down as

$$I_{xy} = \begin{pmatrix} \overline{E_x E_x^*} & \overline{E_x E_y^*} \\ \overline{E_y E_x^*} & \overline{E_y E_y^*} \end{pmatrix}$$

where E_x and E_y are the projections of the electric field along the x and y axes and the overbars denote averaging over time for long enough observation periods. If the determinant of the matrix equals $I^2/4$ (where $I = \overline{|E_x|^2} + \overline{|E_y|^2}$) then the light is

A. completely polarized

B. completely unpolarized

C. 50 % partially polarized

D. 25 % partially polarized

70. For a particle in a one dimensional potential

$$V(x) = -V_o \operatorname{sech}^2(\alpha x)$$

the time period of small oscillations about the equilibrium is

A. $2\pi\sqrt{\dfrac{m}{2V_o\alpha^2}}$

B. $2\pi\sqrt{\dfrac{V_o}{m\alpha^2}}$

C. $2\pi\sqrt{\dfrac{1}{m\alpha^2 V_o}}$

D. $2\pi\sqrt{\dfrac{m}{V_o\alpha^2}}$

71. The Lagrangian of a point particle is

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}\omega^2 x^2 - \alpha x^4 + \beta x \dot{x}^2$$

Its equation of motion is

A. $m\ddot{x} + \beta\dot{x}^2 + \omega^2 x + 4\alpha x^3 = 0$

B. $m\ddot{x} - \beta\dot{x}^2 - \omega^2 x + 4\alpha x^3 = 0$

C. $(m + 2\beta x)\ddot{x} + \beta\dot{x}^2 + \omega^2 x + 4\alpha x^3 = 0$

D. $(m + \beta x)\ddot{x} + \beta\dot{x}^2 + \omega^2 x + 4\alpha x^3 = 0$

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72. A particle of mass m moves in a spherically symmetric potential $V(r) = \frac{1}{2}kr^2$ and has angular momentum L. Which of the following is a constant of motion?

A. $\frac{1}{2}m\left(\frac{d\vec{r}}{dt}\right)^2 + \frac{1}{2}kr^2 + \frac{L^2}{2mr^2}$

B. $\frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}kr^2$

C. $\frac{1}{2}m\left(\frac{d\vec{r}}{dt}\right)^2 + \frac{L^2}{2mr^2}$

D. $\frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}kr^2 + \frac{1}{2}\frac{L^2}{2mr^2}$

73. A particle of mass m and angular momentum $L^2 = 10mV_oR^2$ moves in a potential $V(\vec{r}) = -V_o\left(\frac{3R}{r} + \frac{R^3}{r^3}\right)$, $V_o > 0$. The radius of the stable circular orbit is

A. R

B. $R/2$

C. $2R$

D. $3R$

74. The Poisson bracket of L_x and $\vec{p}^{\,2}$, where $\vec{L} = \vec{r}\times\vec{p}$ denotes the angular momentum, is

A. $2L_x$

B. $2p_x$

C. L_xp_x

D. 0

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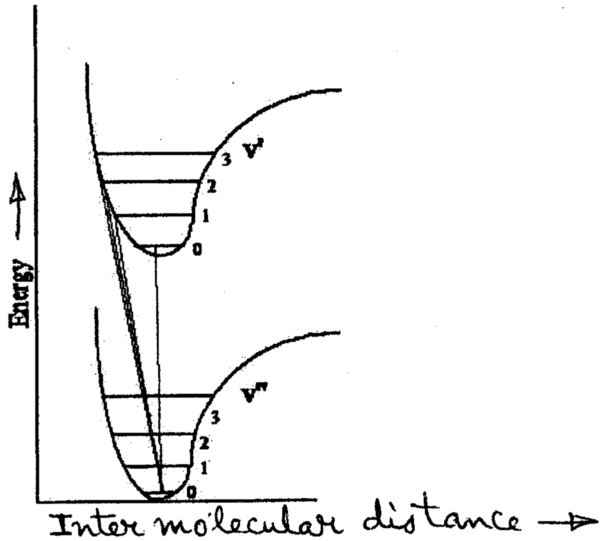
75. Consider a diatomic molecule undergoing a vibrational electronic transition from V'' to V'. Which one of the following is correct?



A. The most intense line will be (0,0) transition

B. The most intense line will be (0,1) transition

C. The most intense line will be (0,2) transition

D. The most intense line will be (0,3) transition