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Entrance Examination : M.Sc. Mathematics, 2011

Hall Ticket Number [| | | | | | |]

Time : 2 hours
Max. Marks. 75

Part A : 25 marks
Part B : 50 marks

Instructions

1. Write your Booklet Code and Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. There is negative marking. In part A a right answer gets **1 mark** and a wrong answer gets **− 0.33 mark**. In part B a right answer gets **2 marks** and a wrong answer gets **−0.66 mark**.
3. Answers are to be marked on the OMR answer sheet following the instructions provided there upon.
4. Hand over the question paper booklet and the OMR answer sheet at the end of the examination.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 50 questions in Part A and Part B together.
8. The appropriate answer should be coloured in either a blue or black ball point or sketch pen. DO NOT USE A PENCIL.

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Part A

1. Statement: All mathematicians are intellectuals.
   Conclusions:
   (1) Raju is not a mathematician so he is not an intellectual
   (2) All intellectuals are mathematicians

   A. Only (1) is correct
   B. Only (2) is correct
   C. Both (1) and (2) are correct
   D. Neither (1) nor (2) is correct

2. For any natural number n, the sum, $\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n} =$

   A. $n2^n$
   B. $n2^{n-1}$
   C. $n2^{n+1}$
   D. none of the above

3. Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be defined as $f(x) = |x|$ then

   A. f is continuous and differentiable
   B. f is continuous but not differentiable
   C. f differentiable but discontinuous
   D. f is discontinuous

4. Let $(a_n), (b_n)$ be two convergent sequences converging to l, m respectively. If we define $c_n = \begin{cases} a_n + m \text{ if } n \text{ is odd,} \\ b_n + l \text{ if } n \text{ is even,} \end{cases}$ then

   A. (c_n) is a Cauchy sequence which is not convergent
   B. (c_n) is bounded but not convergent
   C. (c_n) is a convergent sequence converging to $l + m$
   D. (c_n) has only two convergent subsequences

5. Let G be a group and $a, b \in G$. If $a^{17} = b^{17}$ and $a^{30} = b^{30}$ then

   A. $a = b$
   B. $ab = ba$ and $o(a) \neq o(b)$
   C. $a = b^{-1}$ and $o(a) \neq o(b)$
   D. $o(a) = o(b)$ and $a \neq b$

6. Let $\bar{f}$, ϕ be vector valued and scalar valued functions on $\mathbb{R}^3$ respectively, then

A. curl(grad ϕ) = 0
B. curl(curl $\bar{f}$) = 0
C. grad(div $\bar{f}$) = 0
D. div(grad ϕ) = 0

7. The number of points in the plane equidistant from $P = (-1, 0)$, $Q = (1, 0)$, $R = (0, 1)$ is

A. 0
B. 1
C. 2
D. infinite

8. The number of subgroups of $\mathbb{Z}_{10}$ is

A. 1
B. 2
C. 3
D. 4

9. The number of nontrivial homomorphisms from the cyclic group $\mathbb{Z}_{14}$ to a group of order 7 is

A. 1
B. 2
C. 3
D. 6

10. Let $X = \{0, 1\}$, $Y = \{2, 7\}$, $Z = \{0, 2, 4\}$. Which of them admit group structure

A. only X
B. only X, Z
C. all of them
D. none of them

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11. Two coins whose probabilities of heads showing up are p_1, p_2 are tossed, the probability that at least one tail shows up is

A. $2 - p_1 - p_2$
B. $p_1 p_2$
C. $p_1(1 - p_2)$
D. $1 - p_1 p_2$

12. 2 ones, 2 twos, 1 three and 1 five are to be arranged to get a 6 digit number. The number of different numbers that can be obtained this way is

A. $6\,!$
B. $\frac{6\,!}{2\,!}$
C. $\frac{6\,!}{2\,!\,2\,!}$
D. $\frac{6\,!}{2\,!\,3\,!}$

13. Let $a_n > 0$, $\forall n \in \mathbb{N}$ then consider the statements:
S_1) If $\sum a_n$ converges then $\sum a_n^2$ also converges
S_2) If $\sum a_n^2$ converges then $\sum a_n$ also converges

A. Both S_1 and S_2 are true
B. S_1 is true but S_2 is false
C. S_2 is true but S_1 is false
D. Both S_1 and S_2 are false

14. Let $x \in \mathbb{R}$, $[x]$ denotes the greatest integer less than or equal to x, then consider the statements:
S_1) $[x^2] = [x]^2$
S_2) $|x^2| = |x|^2$

A. Both S_1 and S_2 are true
B. S_1 is true but S_2 is false
C. S_2 is true but S_1 is false
D. Both S_1 and S_2 are false

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15. Let $x \in \mathbb{R} - \{0\}$ then the correct statement is:

A. If $x^2 \in \mathbb{Q}$, then $x^3 \in \mathbb{Q}$
B. If $x^3 \in \mathbb{Q}$, then $x^2 \in \mathbb{Q}$
C. If $x^2 \in \mathbb{Q}$ and $x^4 \in \mathbb{Q}$ then $x^3 \in \mathbb{Q}$
D. If $x^2 \in \mathbb{Q}$ and $x^5 \in \mathbb{Q}$ then $x \in \mathbb{Q}$

16. The number of solutions of $X^5 \equiv 1 (\text{mod} 163)$ in $\mathbb{Z}_{163}$ is

A. 1
B. 2
C. 3
D. 4

17. Let $f : \mathbb{R} \to \mathbb{R}$ be a polynomial such that $f(0) > 0$ and $f(f(x)) = 4x + 1\ \forall x \in \mathbb{R}$, then $f(0)$ is

A. 1/4
B. 1/3
C. 1/2
D. 1

18. Let $f : \mathbb{R} \to \mathbb{R}$ be a polynomial and let (x_n) be a sequence of real numbers converging to 2. Then the sequence $(f(x_n))$ converges to

A. $f(2)$
B. $f(4)$
C. $f(8)$
D. $f(16)$

19. Let V be the vector space of continuous functions on [-1,1] over $\mathbb{R}$. Let $u_1, u_2, u_3, u_4 \in V$ defined as $u_1(x) = x$, $u_2(x) = |x|$, $u_3(x) = x^2$, $u_4(x) = x|x|$ then

A. $\{u_1, u_2\}$ is linearly dependent
B. $\{u_1, u_3, u_4\}$ is linearly dependent
C. $\{u_1, u_2, u_4\}$ is linearly dependent
D. none of the above

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20. The rank of $\begin{bmatrix} 1 & 2 & 1 & -1 \\ -1 & 1 & 2 & 1 \\ 1 & 5 & 4 & -1 \\ -1 & 4 & 5 & 1 \end{bmatrix}$ is

A. 1
B. 2
C. 3
D. 4

21. If $p(x)$ is a polynomial of degree 10^{2011} then $\lim_{x\to\infty} p(x)e^{-x}$

A. is 0
B. is 1
C. is ∞
D. does not exist

22. Let $p = \int_0^1 \sqrt{1-x}\,dx$, $q = \int_0^1 \sqrt{1-x^2}\,dx$ and $r = \int_0^1 \sqrt{1+x}\,dx$. Then

A. $p \leq q \leq r \leq 1$
B. $p \leq q \leq 1 \leq r$
C. $q \leq p \leq 1 \leq r$
D. $1 \leq p \leq q \leq r$

23. Let A be a 4×4 real matrix. Which of the following 4 conditions is not equivalent to the other 3?

A. The matrix A is invertible.
B. The system of equations $Ax = 0$ has only trivial solution.
C. Any two distinct rows u and v of A are linearly independent.
D. The system of equations $Ax = b$ has a unique solution $\forall b \in \mathbb{R}^4$.

24. The set of complex numbers satisfying the equation $z = |z|^2$ is

A. an empty set.
B. a finite set.
C. an infinite set.
D. a line.

25. Let A be a subset of real numbers containing all the rational numbers. Which of the following statements is true?

A. A is countable.
B. If A is uncountable, then $A = \mathbb{R}$.
C. If A is open, then $A = \mathbb{R}$.
D. None of the above statement is true.

Part B

26. Let X be the set of all nonempty finite subsets of $\mathbb{N}$. Which one of the following is not an equivalence relation on X:

A. $A \sim B$ if and only if $\min A = \min B$
B. $A \sim B$ if and only if A, B have same number of elements
C. $A \sim B$ if and only if $A = B$
D. $A \sim B$ if and only if $A \cap B = \phi$

27. Which of the following statements is not true:

A. Every bounded sequence of real numbers has a convergent subsequence
B. If subsequences (x_{2n}) and (x_{3n}) of a sequence (x_n) converges respectively to x and y then $x = y$
C. A monotone sequence of real numbers is convergent if and only if it is bounded
D. A sequence (x_n) of real numbers is convergent if and only if the sequence $(|x_n|)$ is convergent

28. The absolute maximum value of $f(x) = \dfrac{1}{1+|x|} + \dfrac{1}{1+|x-1|}$ on $\mathbb{R}$ is attained at

A. $x = 0$ only
B. $x = 1$ only
C. $x = 0$ and $x = 1$ only
D. no point of $\mathbb{R}$

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29. Which of the following functions is uniformly continuous

A. $f : (0,1) \to \mathbb{R},\ f(x) = \dfrac{e^{\cos x}\sqrt{1+\sinh x}}{2+\tan^2(x^2)}$

B. $f : \mathbb{R} - \{0\} \to \mathbb{R},\ f(x) = x^2 \sin\left(\frac{1}{x}\right)$,

C. $f : \mathbb{R} - \{0\} \to \mathbb{R},\ f(x) = \cos\left(\frac{1}{x}\right)$

D. none of the above

30. If $f : \mathbb{R} \to \mathbb{R}$ then pick up a true statement from the following:

A. If f is continuous then $|f|$ is continuous
B. If f is differentiable then $|f|$ is differentiable
C. If f is integrable then $f(\sqrt{|x|})$ is integrable
D. If f is discontinuous then $|f|$ is discontinuous

31. Let $\bar{f} = (1, f_2(x,y,z), f_3(x,y,z))$ be solenoidal where f_2, f_3 are scalar valued functions. Let S be the unit sphere in $\mathbb{R}^3$ and $\hat{n}$ be unit outward normal. Then $\displaystyle\int_S x\bar{f}.\hat{n}dS =$

A. 0
B. π
C. $4\pi/3$
D. 4π

32. Let $M_3(\mathbb{R})$ be the space of all 3×3 real matrices. Let $V \subset M_3(\mathbb{R})$ be the space of symmetric matrices with trace 0. Then dimension of the quotient space $\dfrac{M_3(\mathbb{R})}{V}$ is

A. 6
B. 5
C. 4
D. 3

33. Let V be the vector space of continuous functions on [-1,1] over $\mathbb{R}$. Which one of the following is a subspace of V

A. $\{f \in V / f$ vanishes at some point in $[-1,1]\}$
B. $\{f \in V / f(0) = 0\}$
C. $\{f \in V / f(x) \neq 0\ \forall x \in [-1,1]\}$
D. $\{f \in V / f(-1) \neq f(1)\}$

34. Let $M_2(\mathbb{R})$ be the space of all 2×2 real matrices, I be the identity matrix in $M_2(\mathbb{R})$. Pick up the correct statement

A. $\exists$ two different matrices $A, B \in M_2(\mathbb{R})$ such that $AB + BA = I$
B. $\exists$ two different matrices $A, B \in M_2(\mathbb{R})$ such that $AB - BA = I$
C. $\exists$ a singular matrix $A \in M_2(\mathbb{R})$ such that $A^2 + I = 0$
D. $\exists A \in M_2(\mathbb{R})$ such that $A^3 \neq 0$ but $A^4 = 0$

35. Let $G = \{1, -1, i, -i\}$ be the group under multiplication. Which of the following statements is true

A. identity map is the only homomorphism from G to G
B. the map $z \to \bar{z}$ is a homomorphism from G to G
C. the map $z \to z^2$ is not a homomorphism from G to G
D. none of the above

36. Each element of a 2×2 matrix A is selected randomly from the set $\{-1, 1\}$ with equal probability. The probability that A is singular is

A. 1/8
B. 2/8
C. 3/8
D. 4/8

37. General solution of $\frac{d^4y}{dx^4} + y = 0$ is

A. $C_1e^x - C_2e^{-x} + C_3\cos x - C_4\sin x$
B. $C_1e^x + C_2xe^{-x} + C_3\cos x + C_4\sin x$
C. $C_1e^x - C_2e^{-x} + C_3\cosh x - C_4\sinh x$
D. $C_1xe^x - C_2e^{-x} + C_3\cos x - C_4\sin x$

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38. Let A_n and B_n, $n \in \mathbb{N}$ be non empty subsets of $\mathbb{R}$ such that $A_1 \supseteq A_2 \supseteq A_3 \supseteq \dots$ and $B_1 \subseteq B_2 \subseteq B_3 \subseteq \dots$. Let the cardinality of A_n be a_n and the cardinality of B_n be b_n. Then the cardinality of

A. $\bigcap_{n=1}^{\infty} A_n$ is $\lim_{n\to\infty} a_n$
B. $\bigcap_{n=1}^{\infty} A_n$ is $\min_n a_n$
C. $\bigcup_{n=1}^{\infty} B_n$ is $\lim_{n\to\infty} b_n$
D. $\bigcup_{n=1}^{\infty} B_n$ is $\max_n b_n$

39. The area bounded on the right by $x + y = 2$, on the left by $y = x^2$ and below by x-axis is

A. 23/6
B. 5/6
C. 17/20
D. 0

40. The distance of the point $(3, -4, 5)$ from the plane $2x + 4y + 6z + 6 = 0$ measured along a line with direction ratios 2,1,3 is

A. $\sqrt{14}$
B. $\sqrt{24}$
C. $\sqrt{30}$
D. $\sqrt{94}$

41. Consider the statements:
S_1) In the set of 2×2 real matrices $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is similar to a diagonal matrix
S_2) If M is a 2×2 real matrix and $M^n = I$ for some $n \in \mathbb{N}$ then $M = I$

A. Both S_1 and S_2 are true
B. S_1 is true but S_2 is false
C. S_2 is true but S_1 is false
D. Both S_1 and S_2 are false

42. Let $f(x) = (x-2)(x-4)(x-6)+2$ then f has

A. all real roots are between 0 and 6
B. a real root between 0 and 1
C. a real root between 6 and 7
D. exactly two roots between 0 and 6

43. For a fixed $y \in [0,\ 1]$, the value of $\int_0^1 [x+y]dx$ is (where for a real number t, $[t]$ is the greatest integer less than or equal to t)

A. 0
B. 1
C. $\frac{1}{2} + y$
D. y

44. Let V be the vector space of all 2×3 real matrices and W be vector space of all 2×2 real matrices. Then

A. there is a one-one linear transformation from $V \to W$.
B. kernel of any linear transformation from $V \to W$ is nontrivial.
C. there is an onto linear transformation from $W \to V$.
D. there is an isomorphism from $V \to W$.

45. Let A be a 2×2 real matrix. Which of the following statements is true?

A. All the entries of A^2 are non-negative.
B. The determinant of A^2 is non-negative.
C. the trace of A^2 is non-negative.
D. all the eigenvalues of A^2 are non-negative.

46. Let $f, g : \mathbb{R} \to \mathbb{R}$ be two differentiable functions. Suppose that $f'(x) > g'(x) > 0$ for $x > 0$. Then

A. $f(x) \geq g(x)$ for all $x > 0$
B. $f(x) \leq g(x)$ for all $x > 0$
C. $f(x) - f(0) \geq g(x) - g(0)$ for all $x > 0$
D. $f(x) - f(0) \geq g(x) - g(0)$ for all x.

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47. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = |x| \sin(x)$. Then

A. f is differentiable at 0 and $f'(0) = 0$.
B. f is differentiable at 0 and $f'(0) = 1$.
C. f is continuous at 0, but not differentiable at 0.
D. f is not continuous at 0, but differentiable at 0.

48. Consider the group $\mathbb{Z}_p \times \mathbb{Z}_p$ under addition. The number of cyclic subgroups of order p is

A. 1
B. $p - 1$
C. $p + 1$
D. $p^2 - 1$

49. Let R be a ring with unity. Then

A. The set of all nonzero elements in R forms a group under multiplication
B. The set of all nonzero invertible elements in R forms a group under multiplication
C. The set of all non zero divisors in R forms a group under multiplication
D. none of the above

50. Let $A, B \subset \mathbb{R}$ and $C = \{a + b / a \in A,\ b \in B\}$. Then the false statement in the following is:

A. If A, B are bounded sets, then C is a bounded set
B. If C is a bounded set, then A, B are bounded sets
C. If $\mathbb{R} - A$, $\mathbb{R} - B$ are bounded sets, then $\mathbb{R} - C$ is a bounded set
D. If $\mathbb{R} - C$ is a bounded set, then $\mathbb{R} - A$, $\mathbb{R} - B$ are bounded sets