

1. A microwave transmitter on one bank of a wide lake and at a height  $a$  above water level transmits microwaves of wavelength  $\lambda$  towards a receiver on the opposite bank, a distance  $x$  above the water level. Microwaves reflected from the water surface interfere with those arriving directly at the receiver. The width of the lake  $D$  is very large as compared to the heights  $a$  and  $x$  of the transmitter and receiver. For what values of  $x$  will the signal at the receiver be maximum? 10
2. Light of wavelength 600 nm falls normally on a grating. Two adjacent maxima are obtained for  $\sin \theta_1 = 0.2$  and  $\sin \theta_2 = 0.3$ . The 4<sup>th</sup> order maximum is missing.  
 (a) What is the separation between adjacent slits of the grating?  
 (b) What is the smallest slit width the grating can have?  
 (c) Which orders of intensity maxima are produced by the grating, assuming the values obtained in (a) and (b)? 10
3. A beam of partially polarized light can be considered to be a mixture of plane polarized and unpolarized light. Suppose we send such a beam through a polarizing filter and then rotate the filter by  $360^\circ$  always keeping it normal to the beam. If the transmitted intensity during this rotation varies maximum by a factor of 5, what fraction of the intensity of the original beam is associated with its polarized component? 10
4. A radioactive nucleus at rest had a mass  $M$ . When moving with a total energy of  $E$ , the nucleus emits a  $\gamma$ -ray in the direction of its motion and drops to its stable non-radioactive state at rest with mass  $m$ . Find an expression for the total energy  $E$  of the nucleus. Energy of the outgoing  $\gamma$ -ray should not appear anywhere in this expression. 10
5. In an inertial frame  $S$ , a red light flashes on its  $x$ -axis at position  $x_R = 3.0\text{m}$  at time  $t_R = 1.0 \times 10^{-9}$  s, and a blue light flashes at  $x_B = 5.0\text{m}$  at time  $t_B = 9.0 \times 10^{-9}$  s. Another frame  $S'$  has its axes parallel to those of  $S$  and its origin at the same point as that of  $S$  at  $t = t' = 0$ , but it moves relative to  $S$  with a constant velocity in the  $x$ -direction. The two flashes are observed to occur in  $S'$  at the same position. Find  
 (a) the relative speed between  $S$  and  $S'$   
 (b) the location of the two flashes in  $S'$   
 (c) the timing of the two flashes in  $S'$  10
6. It is desired to have a single-mode step index fibre, with a numerical aperture of 0.20, to be used at 820 nm. If the core material has a refractive index of 1.458, what should be the refractive index of the cladding material and the core diameter of the fibre?  
 If this fibre has a loss of 1.5 dB/km, what minimum power must be launched into the fibre to maintain an output power level of at least  $0.3 \mu\text{W}$  at the end of a 12 km long fibre? 10
7. A plane electromagnetic wave with wavelength 3.0 m, travels in vacuum in the positive  $X$ -direction with its electric field  $\vec{E}$  of amplitude 300 V/m, directed along the  $Y$ -axis.  
 (a) What is the frequency of the wave?  
 (b) What are the direction and amplitude of the magnetic field associated with the wave?  
 (c) What are the magnitudes of angular frequency and wave vector associated with this wave?  
 (d) What is the time-averaged rate of energy flow in  $\text{W/m}^2$  associated with this wave?  
 (e) If the wave falls on a perfectly absorbing sheet of area  $2.0 \text{ m}^2$ , at what rate is momentum delivered to the sheet? 10
8. Write down the expressions for electric and magnetic field vectors for a plane monochromatic electro-magnetic wave travelling in vacuum. Show that  
 (a) the waves are transverse in nature  
 (b) the electric and magnetic field vectors oscillate in phase and are mutually perpendicular  
 Also find the relation between the magnitudes of the electric and magnetic field vectors. 10