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Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, June 2009
(2003 Scheme)
03.501: ENGINEERING MATHEMATICS – IV (CMNPHETARUFB)

Time : 3 Hours Max. Marks : 100

Instruction: *Answer* ***all*** *questions from Part –* ***A and one*** *question from* ***each*** *Module.*

PART – A

1. Using Cauchy Reimann Equations show that $f(z) = |z|^2$ is not analytic at any point.

2. Show that f(z) is analytic and
   i) Real f (z) is constant
   ii) Im.f(z) is constant, then f(z) is a constant.

3. Show that under the transformation $w = \frac{1}{z}$ all circles in the z plane is transformed in to circles or straight lines in the w plane.

4. Show that $\int_C \frac{e^z}{z} dz = 2\pi i, \quad c : |z| = 1$.

5. Expand $\frac{1}{z^2 - 3z + 2}$ the region $0 < |z - 1| < 1$.

6. Define fixed point and critical point of a bilinear transformation. Find the fixed point of $w = \frac{5 - 4z}{4z - 2}$.

P.T.O.

7. Evaluate $\int_C \tan z \, dz$ where c is the circle |z| = 2.

8. A random sample of 500 apples was taken from a large consignment and 60 were found to be bad. Obtain a 95% limits for percentage of bad apples in the consignment.

9. A random variable X has the following probability function :

Values of X x :	0	1	2	3	4	5	6	7
p(x) :	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(1) find k

(2) evaluate $p[X \le 6]$, $p[X \ge 6]$, $p[3 < X \le 6]$.

10. During war, 1 ship out of 9 was sunk of on an average in making a certain voyage. What was the probability that exactly 3 out of a convoy of 6 ships would arrive safely ?

PART – B

MODULE – I

11. a) Determine an Analytic function whose real part is e^{2x} (x cos 2y – y sin 2y).

b) If f (z) is an Analytic function prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|\text{Re}\, f(z)|^2 = 2|f'(z)|^2$$

c) Determine the region in the w plane into which the region $\frac{1}{2} \le x \le 1$ and $\frac{1}{2} \le y \le 1$ is mapped by the transformation $w = z^2$.

12. a) If f (z) = u + iv is an analytic function and find f (z) if $u + v = \frac{x}{x^2 + y^2}$ when f (1) = 1.

b) Find the bilinear transformation which maps the point z = 1, i, –1 on to the points w = i, 0, – i. Hence find the image of |z| < 1.

c) Find the image of the circle |z – 3| = 5 under the transformation $w = \frac{1}{z}$.

MODULE – II

13. a) Integrate f (z) = $x^2 + ixy$ from A (1, 1) to B (2, 4) along the curve x = t, y = t^2.

b) Expand $\frac{1}{z^2 - 4z + 3}$ as a Laurent's series in 1< |z|< 3.

c) Evaluate using Residue theorem $\int_c \frac{\sin \pi z^2 + \cos \pi z^2 dz}{(z-1)^2(z-2)}$ where c : |z| = 3.

14. a) Show that $\int_0^{2\pi} \frac{d\theta}{(5-3\cos\theta)^2} = \frac{5\pi}{12}$

b) Evaluate $\int_0^{\infty} \frac{dx}{1+x^4}$.

MODULE – III

15. a) Find the mean and variance of the Binomial distribution.

b) Fit a parabola to the data :

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

c) For a normally distributed variate x with mean 1 and S.D. 3, find the probability that $3.43 \leq x \leq 6.19$.

16. a) In two colleges affiliated to a university 64 out of 200 and 48 out of 250 candidates failed in an examination.

If the percentage failure in the university is 18%, examine whether the colleges differ significantly.

b) Out of 800 families of 5 children each, how many would you expect to have
1) 3 boys 2) 5 girls ?

c) If X is a Poisson variate such that P[X = 2] = 2P [X = 4] + 90 P [X = 6] find the S.D.