Reg. No. ....

(2 pages)

K 5101

Name ....

FIRST YEAR B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2005

Part III—Group I—Mathematics

Paper I—CALCULUS, ANALYTIC GEOMETRY AND TRIGONOMETRY

Time : Three Hours | Maximum : 65 Marks

Maximum marks that can be earned from each unit is 13.
Each question carries 5 marks.

Unit I

1. If $y = (x^2 - 1)^n$, show that $(x^2 - 1)\, y_{n+2} + 2\, x\, y_{n+1} - n\,(n+1)\, y_n = 0$.

2. State Rolle's theorem. Verify the theorem for the function $f(x) = x\,(x + 3)\, e^{-x/2}$.

3. Using Maclaurin's series, prove that $\tan^{-1} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dots$

4. State Euler's theorem on homogeneous functions and apply it to show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$, where $u = \sin^{-1}\dfrac{x^2 + y^2}{x + y}$.

Unit II

5. Find the radius of curvature of the curve $x^3 + y^3 = xy$ at the point $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$.

6. Find the evolute of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

7. Show that the points of inflexion of the curve $y^2 = (x - m)^2\,(x - n)$ lies on the line $3\,x + m = 4\,n$.

8. Find the envelope of the family of straight lines $x \cos \alpha + y \sin \alpha = l \sin \alpha \cos \alpha$, the parameter being α.

Unit III

9. Define polar of a point with respect to a conic. Show that the locus of poles of normal chords of the parabola $y^2 = 4\,ax$ is $(x + 2\,a)\, y^2 + 4\,a^3 = 0$.

10. Show that the locus of middle points of chords of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ touching the circle $x^2 + y^2 = c^2$ is $\left(\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}\right)^2 = c^2\left(\dfrac{x^2}{a^4} + \dfrac{y^2}{b^4}\right)$.

11. Prove that the tangents at the ends of a pair of conjugate diameter, of an ellipse form a parallelogram of constant area.

12. The asymptotes of a hyperbola are parallel to the lines $x + 2y - 7 = 0$ and $3x + 2y - 1 = 0$ and the centre is at (1, 1). If the hyperbola passes through (2, 5), find its equation. Find also the equation to the conjugate hyperbola.

Unit IV

13. Show that the general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a parabola if $ab - h^2 = 0$.

14. Show that $5x^2 - 6xy + 5y^2 + 22x - 26y + 29 = 0$ represents an ellipse and find its centre. Obtain the equation to the ellipse referred to its centre as new origin and axes parallel to the original axes.

15. Find the polar equation of tangent at any point of the conic $\frac{l}{r} = 1 + e\cos\theta$.

16. Find the polar equation of a circle whose centre is (γ_1, θ_1) and radius q. Deduce the equation of the circle passing through the point.

Unit V

17. Expand $\cos^6\theta \sin^2\theta$ in a series of cosines of multiples of θ.

18. Resolve into real factors $x^7 + 1$.

19. Find the sum to infinity of the series $\sin\alpha + \frac{1}{2}\sin(\alpha + \beta) + \frac{1 \cdot 3}{2 \cdot 4}\sin(\alpha + 2\beta) + \ldots$

20. If $\tan(A + iB) = x + iy$, prove that $x^2 + y^2 + 2x\cot 2A = 1$.