

FIRST YEAR B.Sc. DEGREE EXAMINATION, APRIL/MAY 2005

Part III—Mathematics

Subsidiary Paper I—MATHEMATICS

(Common for all subsidiary subjects)

Time : Three Hours

Maximum : 70 Marks

A maximum of 14 marks will be awarded from each unit.
All questions carry 5 marks each.

Unit I

- 1. Find the sum of infinity of the series 1 - 3/4 + 3.5/4.8 - 3.5.7/4.8.12 + ...
2. If n is large show that (1 + 1/n)^n = e(1 - 1/2n + 11/24n^2) approximately.
3. If alpha, beta, gamma are the roots of the equation x^3 + px + q = 0, find the equation whose roots are (alpha - beta)^2, (beta - gamma)^2, (gamma - alpha)^2.
4. Solve the biquadratic equation x^4 - 4x^3 + 7x^2 - 6x + 2 = 0.

Unit II

- 5. If tan(x + iy) = u + iv, prove that u/v = sin 2x / sinh 2y.
6. Sum to infinity the series c sin alpha + c^2/2! cos 2 alpha + c^3/3! cos 3 alpha + ...
7. Transform the equation 6x^2 + 24xy - y^2 = 0 into another in which the xy term is absent.
8. The asymptotes of a hyperbola are parallel to the lines 2x + 3y = 0 and 3x - 2y = 0 and its centre is at (1, 2). Find its equation if it passes through the point (5, 3). Find also the equation to the conjugate hyperbola.

Unit III

- 9. Expand sin x as an infinite series.
10. Evaluate lim (tan x)^(tan 2x) as x approaches pi/4.
11. Find the maxima and minima of the function x^3 - 18x^2 + 96x + 4.
12. Find the radius of curvature of the curve x^(1/2) + y^(1/2) = 1 at the point (1/4, 1/4).

Turn over

Unit IV

13. Find the area lying above the x -axis and included between the circle $x^2 + y^2 - 2x = 0$ and the parabola $y^2 = x$.
14. Find the area of the surface formed by the rotation of the curve $y^2 = 8x$ about the x -axis from $x = 2$ to $x = 7$.
15. Evaluate $\iint (x^2 + y^2) dx dy$ over the region for which $x \geq 0, y \geq 0$ and $x + y \leq 1$.
16. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta$.

Unit V

17. Show that $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal.
18. Test for consistency and solve :
- $$\begin{aligned} x + 2y - z &= 3 \\ 3x - y + 2z &= 1 \\ 2x - 2y + 3z &= 2 \end{aligned}$$
19. Find the eigen vectors of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$.
20. If $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$, find A^4 using Cayley-Hamilton theorem.