(Please write your Exam Roll No.) Exam Roll No. ....

END-TERM EXAMINATION

THIRD SEMESTER [B.TECH] – DECEMBER-2007

Paper Code:ETEC203 (Batch-2004-2006)	Subject: Signal and Systems
Paper ID: 28203	
Time : 3 Hours	Maximum Marks : 75

Note: Attempt all questions. Question no. 1 is compulsory. Internal choice is indicated.

Q.1 (a) If $x_1(t)$ and $x_2(t)$ are periodic signals with time periods T_1 and T_2 respectively, then under what conditions is the sum $x(t) = x_1(t) + x_2(t)$ periodic and what is the fundamental time period of x(t) if it is periodic. (3)

(b) The impulse response of a discrete time LT1 system is given by $h[n] = \alpha^n u[n]$. Compute the unit step response of the system. (3)

(c) What are the discrete-time Fourier series coefficients of the sequence x[n] = sin $\frac{2\pi}{9} > n$. (3)

(d) Show that $\int_{-\infty}^{\infty} \text{sinc}^2(x)\, dx = 1$

and $\int \text{sinc}\,(x)\, dx = 1$ (4)

(e) If x[n] is add, then show that its z-transform x[z] has a zero at z = 1. (3)

(f) Let x(s) and R be the Laplace transform and ROC of x(t). What will be the Laplace transform and ROC of y(t) = -t x(t). (3)

(g) A continuous time LTI system which is also stable and causal is described by the differential equation. $\frac{dy(t)}{dt} + 5y(t) = 24x(t)$, where y(t) is the output when the input is x(t). What is the final value s (∞) of the step response s(t) of this system. (3)

(h) Let x(t) be a low pass signal with band width WHz. Compute the Nyquist rate for
(i) $x(t) + x(t-1)$ (ii) $x^2(t)$ (3)

Q.2 (a) The output y [n] and input x [n] of a system are related as $y[n] = \sum_{k=-\infty}^{\infty} 2^{k-n} x[k+1]$ (6.5)

(i) Find out whether the system is linear, time invariant, stable and causal.
(ii) Find the impulse response of the system.

(b) Compute the unit step response of an LT1 system whose impulse response is given be $h(t) = \begin{cases} e^{2t} & t < 0 \\ e^{-3t} & t \geq 0 \end{cases}$. (6)

OR

(c) The output y(t) and input x (t) of a continuous time system are related as $y(t) \frac{1}{T} \int_{-T/2}^{+T/2} x(z)\, dz$. (6.5)

(i) Is this system linear, time invariant, causal and stable?
(ii) Find and sketch the impulse response of the system.

(d) Let x[n] be a discrete time signal, and let $y_1[n] = x[2n]$ and $y_2[n] = \begin{cases} x[n/2] & n \text{ is even.} \\ 0 & n \text{ is odd.} \end{cases}$ (6)

Determine whether the following statements are true or false. Give reasons.
(i) If x[n] is periodic, $y_1[n]$ is also periodic.
(ii) If $y_2[n]$ is periodic, x[n] is also periodic.
(iii) If $y_1[n]$ is periodic, x[n] is also periodic.

Q.3 (a) Let p(t) be a signal such that p(t) = 0 for |t| > T/4. Define the periodic signal $x(t) = \sum_{n=-\infty}^{\infty} p(t - nT)$. Determine the Fourier series coefficient of x(t) in terms of the Fourier transform of p(t). (6.5)

(b) For an aperiodic discrete time signal x[n], show that, $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |x(jw)|^2 dw$, where x(w) is the DT transform of x [n]. (6)

OR

(c) A signal x[n] is periodic with time period N, and discrete time Fourier coefficients a_k satisfy the following: - (6.5)

(i) x [n] is real (ii) $a_1 = 1$

(iii) $\sum_{n=0}^{N-1} |x[n]|^2 = 3N$ (iv) $\sum_{n=0}^{N-1} x(n) = N$ find x [n]

(d) Let the Fourier transform of n(t) by x(w). Find out the Fourier transform of the following signals in terms of x(w). (6)

(i) $y_1(t) = \text{Re}\{x(t)\}$ (ii) $y_2(t) = x(1-t) + x(-1-t)$

(iii) $y_3(t)$ = even part of { x(t) – cos wot}

Q.4 (a) Let $x_1(w)$ and $x_2(w)$ be the Fourier transforms of $x_1(t)$ and $x_2(t)$ respectively.

$x_1(w) = 0 \quad |w| \geq w_1$

$x_2(w) = 0 \quad |w| \geq w_2$

Compute the Nyquist sampling rate of y (t) = $x_1(t).x_2(t)$ and p(t) = $x_1(t) \cdot x_2(t)$ → convolution. (6.5)

(b) The impulse response of a LT1 system is defined by $h(t) = \begin{cases} \exp(-t) & t > 0 \\ 1/2 & t = 0. \\ 0 & t < 0 \end{cases}$

Determine the phase delay and group delay of the system. Interpret the results (6)

OR

(c) Determine whether each of the following statements is true or false. (6.5)

(i) The signal $x(t) = u(t + T_0) - u(t - T_0)$ can undergo impulse train sampling without aliasing if the sampling period $T < 2T_0$.

(ii) The signal x(t) with Fourier transform. $X(jw) = u(w - w_0) - u(w - w_0)$ can undergo impulse train sampling without aliasing if the sampling period. $T < \pi/w_0$.

(iii) The signal x(t) with Fourier transform $x(jw) = u(w) - u(w - w_0)$ can undergo impulse train sampling without aliasing provided that the sampling period $T < 2\pi/w_0$.

(d) Which of the following systems is a linear phase system. Justify your answers. (6)

(i) $H(w) = \frac{1}{1 + jw}$ (ii) $H(w) = \frac{1}{(1 + jw)^2}$

(iii) $H(w) = \frac{1}{(1 + jw)(2 + jw)}$

Q.5 (a) The following information about a discrete time signal n(n) is given: - (6.5)

(i) x [n] is real and right sided. (ii) x [z] has exactly two poles.

(iii) x [z] has two zeroes at the origin. (iv) x [z] has a pole at $z = \frac{1}{2} e^{j\pi/3}$

(v) x [1] = 8/3

Determine x(z) and specify it's ROC.

(b) Let the Laplace transform of x(t) be x(s). What are the properties of the ROC of x(s) is x(s) is a rational function of 's'. (6)

OR

(c) Determine the forced and natural response of an LT1 system described by the following differential equation and initial conditions. (6.5)

$$\frac{d}{dt} y(t) + 10\, y(t) = 10x(t) \quad y(0^-) = 1$$

y(t) is o/p and x(t) is input = u(t).

(d) A signal x[n] is even and has a rational transfer function (6)

(i) What constraints must the poles of such a signal satisfy?

(ii) If $x[z] = \frac{2 - (17/4)z^{-1}}{(1 - z^{-1/4})(1 - 4z^{-1})}$. Determine the ROC and find x[n].

****