(Please write your Exam Roll No.) Exam Roll No. 16915628 07

END TERM EXAMINATION

SECOND SEMESTER [B.TECH.] - MAY-JUNE 2009

Paper Code: ETMA-102	Subject: Applied Mathematics-II
Paper ID: 99102	(Batch: 2004-2008)

Time : 3 Hours   Maximum Marks :75

Note: Attempt one question from each unit. Q.No.1 is compulsory.

Q.1 (a) If $\dfrac{x^2}{a^2+u}+\dfrac{y^2}{b^2+u}+\dfrac{z^2}{c^2+u}=1$, prove that

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right).$$ **(3)**

(b) For the function, $\varphi(x,y)=\dfrac{x}{x^2+y^2}$, find the magnitude of the directional derivative along a line making an angle 30° with the positive x-axis at (0, 2). **(3)**

(c) Find the inverse Laplace transform of $\cot^{-1}\left(\dfrac{S}{2}\right)$. **(3)**

(d) Apply Green's thm. in plane to evaluate $\int_C\left[\left(3x^2-8y^2\right)dx+\left(4y-6xy\right)dy\right]$, where C is the boundary of the region defined by $y=\sqrt{x}$ & $y=x^2$. **(3)**

(e) Show that when $|z+1|<1$, $Z^{-2}=1+\sum_{n=1}^{\infty}(n+1)(Z+1)^n$. **(4)**

(f) Find Laplace transform of $f(t)=\begin{cases} t^2 & ; \quad 0<t<2 \\ t-1 & ; \quad 2<t<3 \\ 7 & ; \quad t>3 \end{cases}$ **(3)**

(g) Expand $\tan^{-1}\dfrac{y}{x}$ in the neighbourhood of (1, 1). **(3)**

(h) Prove that $\oint_C (z-a)^n dz=0$ (n is an integer ≠ -1), where C is the circle $|Z-a|=r$. **(3)**

UNIT-I

Q.2 (a) If $u=\sin^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=-\frac{\sin u\cos 2u}{4\cos^3 u}.$$ **(6)**

(b) If $w=f(x,y)$, $x=r\cos\theta$, $y=r\sin\theta$, show that $\left(\dfrac{\partial w}{\partial r}\right)^2+\dfrac{1}{r^2}\left(\dfrac{\partial w}{\partial \theta}\right)^2=\left(\dfrac{\partial f}{\partial x}\right)^2+\left(\dfrac{\partial f}{\partial y}\right)^2$. **(3.5)**

(c) If $x^2+y^2+z^2-2xyz=1$, show that $\dfrac{dx}{\sqrt{1-x^2}}+\dfrac{dy}{\sqrt{1-y^2}}+\dfrac{dz}{\sqrt{1-z^2}}=0$. **(3)**

Q.3 (a) Transform the equation $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$ into polar coordinates. **(6.5)**

(b) Examine the following function for extreme values $x^4+y^4-2x^2+4xy-2y^2$. **(6)**

P.T.O.

UNIT-II

Q.4 (a) Show that the function $f(z) = e^{-z^{-4}}$, $z \neq 0$ and $f(0) = 0$ is not analytic at $z = 0$, although Cauchy-Riemann equations are satisfied at this point. **(6.5)**

(b) If $w = z + \frac{a^2}{z}$, prove that, when z describes the circle $x^2 + y^2 = a^2$, w describes a straight line and find its length. Also prove that if z describes the circle $x^2 + y^2 = b^2$, where $b > a$, w describes an ellipse. **(6)**

Q.5 (a) Evaluate by the method of complex variables, the integral $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(1+x^2)^3}$. **(6)**

(b) State and prove Cauchy's Integral formula. Hence evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where C is the circle $|Z| = 2$. **(6.5)**

UNIT-III

Q.6 (a) Apply Stoke's theorem to evaluate $\int_C (ydx + zdy + xdz)$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. **(6.5)**

(b) Using Divergence theorem, evaluate $\iint_S \vec{F}.\vec{ds}$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ & S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **(6)**

Q.7 (a) Find the work done in moving a particle once round the circle $x^2+y^2=9$ in the xy-plane if the field of force is $\vec{F} = (2x - y - z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$. If possible, find its scalar potential. **(6)**

(b) Find the values of a, b, c for which the vector $\vec{V} = (x + y + az)\hat{i} + (bx + 3y - z)\hat{j} + (3x + cy + z)\hat{k}$ is irrotational. **(3.5)**

(c) Evaluate $\iint_S \vec{r}.\hat{n}\,dS$, where S is a closed surface. **(3)**

UNIT-IV

Q.8 (a) Solve by using Laplace Transform, $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = \frac{17}{2}\sin 2t$, given $y = 2$ and $\frac{dy}{dt} = -4$, when $t = 0$. **(6.5)**

(b) Find the Laplace transform of the triangular wave function of period 2c given by $f(t) = \begin{cases} t, & 0 < t < c \\ 2c - t, & c < t < 2c \end{cases}$ **(6)**

Q.9 (a) Apply convolution theorem to evaluate $L^{-1}\left(\frac{s^2}{s^4 - a^4}\right)$. **(4)**

(b) Evaluate $L\left(\int_0^t \frac{e^t \sin t}{t}dt\right)$. **(4.5)**

(c) Find the inverse Laplace Transform of $\frac{S}{S^4 + 4a^4}$. **(4)**

****