B. Tech Degree IV Semester Examination, April 2009

IT/CS/EC/CE/ME/SE/EB/EI/EE/FT
401 ENGINEERING MATHEMATICS III
(2006 Scheme)

Time : 3 Hours | Maximum Marks : 100

PART - A
(Answer ***ALL*** questions)

(8 x 5 = 40)

I. (a) Show that the function $f(z) = \sin z$ is analytic.

(b) What do you mean by conjugate harmonic function? Verify whether the function $e^x \sin y$ is harmonic.

(c) State and prove Canchy's integral formula.

(d) Define the residue of a function at an isolated singularity and determine the poles of $\dfrac{z^2 - 2z}{(z+1)^2(z^2+1)}$ and the residue at each pole.

(e) Form the partial differential equation by eliminating the arbitrary constants in $z = (x-a)^2 + (y-b)^2 + 1$.

(f) Solve $p^2 + q^2 = x^2 + y^2$.

(g) Derive one dimensional heat equation.

(h) Solve the equation $\dfrac{\partial u}{\partial x} = 4\dfrac{\partial u}{\partial y}$, $u(0,y) = 8e^{-3y}$ by the method of separation of variables.

PART – B

(4 x 15 = 60)

II. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not regular at the origin, eventhough Canchy – Riemann equations are satisfied at the origin. (5)

(b) Find the analytic function $w = u + iv$, given that $v = e^{2x}(x\cos 2y - y\sin 2y)$. (5)

(c) Find the equation of the orthogonal trajectories of the family of curves given by $3x^2y + 2x^2 - y^3 - 2y^2 = a$, where a is an arbitrary constant. (5)

OR

III. (a) What do you mean by conformal mapping? Also discuss about
   (i) Translation
   (ii) Magnification and rotation (5)

(b) Discuss the transformation about $w = \sin z$. (5)

(c) Show that the image of the hyperbola $x^2 - y^2 = 1$ under the transformation $w = \dfrac{1}{z}$ is the lemniscate $R^2 = \cos 2\phi$. (5)

(Turn Over)

IV. (a) Verify Canchy's theorem for the integral of z^3 taken over the boundary of the rectangle with vertices $-1, 1, 1+i, -1+i$. (5)

(b) Find the Lausent's series expansion of $\frac{z^2-1}{z^2+5z+6}$ about $z=0$ in the region $2<|z|<3$. (5)

(c) Using Residue theorem, evaluate $\int_c \frac{3z^2+2}{(z-1)(z^2+9)}dz$ where $C:|z-2|=2$. (5)

OR

V. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 \quad dx}{(x^2+a^2)\quad(x^2+b^2)}$ using contour integration where $a>b>0$. (8)

(b) Evaluate $\int_0^{\infty} \frac{\sin mx}{x}dx$ using contour integration where $m>0$. (7)

VI. Solve –

(i) $p(1+q)=qz$ (4)

(ii) $z^2(p^2x^2+q^2)=1$ (5)

(iii) $(x^2+y^2+yz)p+(x^2+y^2-xz)q=z(x+y)$ (6)

OR

VII. Solve –

(i) $(D^3-3D^2D^1+4D'^3)z=e^{x+2y}$ (4)

(ii) $(D^2-3DD^1+D'^2)z=\sin x\cos y$ (5)

(iii) $(D^3-7DD'^2-6D'^3)=x^2y+\sin(x+2y)$ (6)

VIII. (a) A string is stretched and fastened to two points ℓ apart. Motion is started by displacing the string in the form $y=a\sin(\pi x/\ell)$ from which it is released at time $t=0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t)=a\sin(\pi x/\ell)\cos(\pi ct/\ell)$. (8)

(b) Obtain D'Alembert's solution of the wave equation by the method of separation of variables. (7)

OR

IX. (a) A string is stretched and fastened to two points $x=0$ and $x=\ell$ apart. Motion is started by displacing the string into the form $y=k(\ell x-x^2)$ from which it is released at time $t=0$. Find the displacement of any point on the string at a distance of x from one end at time t. (8)

(b) Obtain solution of Laplace's equation over a rectangular region by the method of separation of variables. (7)

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