B.Tech. Degree I & II Semester (Combined) Examination, June 2008

IT/CS/EC/CE/ME/SE/EB/EI/ EE/FT 101 ENGINEERING MATHEMATICS I

(2006 Scheme)

Time: 3 Hours   Maximum Marks: 100

PART A

(Answer all questions)

(8 x 5 = 40)

I a) Solve $\frac{d^4y}{dx^4} - y = e^{-x} Cos\, x$.

b) Test the convergence of the series $\sum_{2}^{\infty} \frac{1}{n^2 \log_e^n}$.

c) If $x^x y^y z^z = C$ find $\frac{\partial^2 z}{\partial x \partial y}$.

d) Find the area bounded by $y^2 = \frac{4a^2(2a-x)}{x}$ and its asymptote.

e) Solve $(x^2 - 3xy + 2y^2)dx + x(3x-2y)dy = 0$.

f) Verify whether the functions $u = \frac{x+y}{1-xy}$ $V = \tan^{-1} x + \tan^{-1} y$ are functionally dependent and if so find the relation between them.

g) Show that $\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{19^2} - - - - -$.

h) Prove that $\lceil n = \int_0^1 (\log \frac{1}{x})^{n-1} dx (n > 0)$.

PART B

(15 x 4 = 60)

II Solve the following differential equations :

i) $(1 + e^{x/y})dx + e^{x/y}(1 - x/y)dy = 0$ (4)

ii) $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$ (6)

iii) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2Sin^2 x + 2x^2$ (5)

OR

III a) An RL circuit has an emf of 5V, a resistance of 50 Ω, an inductance of 1H and no initial current. Find the current in the circuit at any time t. (7)

b) Solve $\frac{dx}{dt} + 4x + 3y = t$

$\frac{dy}{dt} + 2x + 5y = e^t$, given that $x = y = 0$ when $t = 0$. (8)

IV a) Expand $Sinx$ in powers of $(x - \pi/2)$. Hence find the value of $Sin\, 91^\circ$ correct to four decimal places. (6)

b) Find the interval of convergence of the series $x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} - - - - - - - -$. (5)

c) Define conditional convergence and absolute convergence of series with examples. (4)

OR

(Turn Over)

V a) Discuss the convergence of the power series $\sum_{1}^{a} \frac{(n!)^2}{(2n)!} x^{2n}$. (7)

b) If $y = (Sin^{-1}x)^2$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$. Hence find the value of y_n at $x = 0$. (8)

VI a) The focal length of a mirror is given by the formula $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$. If equal errors δ are made in the determination of u and v, show that the relative error in the focal length is given by $\left[\frac{1}{u} + \frac{1}{v}\right]\delta$. (5)

b) If $x = u + v + w,\ y = uv + vw + wu,\ z = uvw$ and f is a function of x, y, z prove that $x\frac{\partial f}{\partial x} + 2y\frac{\partial f}{\partial y} + 3z\frac{\partial f}{\partial z} = u\frac{\partial f}{\partial u} + v\frac{\partial f}{\partial v} + w\frac{\partial f}{\partial w}$. (5)

c) Find the expansion of the function $e^x \log(1+y)$ in a Taylor series in the neighbourhood of (0,0). (5)

OR

VII a) If $u = xf(y/x) + g(y/x)$ prove that $x\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$. (6)

b) In a plane triangle ABC, find the maximum value of $CosACosBCosC$. (6)

c) Find the total differential coefficient of $x^2 y$ with respect to x when x and y are connected by the relation $x^2 + xy + y^2 = 1$. (3)

VIII a) Find the area of the surface of revolution formed by revolving the loop of the curve $9ay^2 = x(3a-x)^2$ about x - axis. (5)

b) Change the order of integration in $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2 + y^2}$ and hence evaluate the same. (5)

c) Evaluate $\iiint \frac{dx\,dy\,dz}{\sqrt{1 - x^2 - y^2 - z^2}}$ over the first octant of the unit sphere by transforming into spherical polar co-ordinates. (5)

OR

IX a) Find the area included between the cardioids $r = a(1 + Cos\theta)$ and $r = a(1 - Cos\theta)$. (5)

b) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (7)

c) Express $\int_0^{\pi/2} \sqrt{Cot\theta}\, d\theta$ in terms of gamma function. (3)

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