

B. Tech Degree I & II Semester (Combined) Examination June 2008

IT/CS/EC/CE/ME/SE/EB/EI/EE 102
ENGINEERING MATHEMATICS II
(2000 Scheme)

Time : 3 Hours

Maximum Marks : 100

(All questions carry EQUAL marks)

I. (a) Discuss the convergence of

(i)
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$$

(ii)
$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots \infty$$

(b) Prove that the following series converges absolutely $\frac{\cos x}{1^2} + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \dots \infty$.

(c) Find the first three non zero terms of the Taylor series expansion of $e^x \sin x$ about $x = 0$.

OR

II. (a) Discuss the convergence of

(i)
$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \infty$$

(ii)
$$1 + x + x^2 + \dots \infty$$

(b) Find the Taylor series expansion of $\cos x$ about $x = 0$.

(c) If $y = \sin^{-1} x$, show that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$.

III. (a) Solve the following system of equation by Gauss Elimination method :

$$7x + 6y - 5z = 30$$

$$3x - 4y + z = 0$$

$$x + 2y - 3z = 10$$

(b) Using Cayley Hamilton theorem obtain the inverse of

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

OR

IV. (a) Find the eigen values of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ and find eigen vector corresponding

to the smallest eigen value.

(Turn Over)

- (b) Express the following matrix as the sum of a symmetric and a skew-symmetric matrix :

$$\begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$$

V. (a) Solve $\frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} + 12y = e^{3x} + x \sin x$

(b) Solve $\frac{dx}{dt} - y = e^t$

$$\frac{dy}{dt} + x = \sin t \quad \text{given that } x = 0, y = 0$$

when $t = 0$

OR

VI. (a) Solve $\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = 2(1 + x - x^2)$

(b) Solve $\frac{d^3 y}{dx^3} + y = \sin 2x + x^2 e^{-x}$

VII. (a) Find the Laplace transforms of (i) $\sin 2t \cos 2t$ (ii) $t^3 e^{-3t}$

(b) Find the inverse Laplace transform of (i) $\frac{4s+5}{(s-1)^2(s+2)}$ (ii) $\frac{s}{(s-3)(s^2+4)}$

OR

VIII. (a) Solve using Laplace transform $\frac{d^4 y}{dt^4} - k^4 y = 0$ given that

$$y(0) = 1, y'(0) = y''(0) = y'''(0) = 0.$$

(b) Use convolution theorem to find $L^{-1} \left\{ \frac{1}{s(s^2+4)} \right\}$

IX. (a) Find the values of the constants a, b, c for which the vector $\vec{f} = (x + y + az)\vec{i} + (bx + 3y - z)\vec{j} + (3x + cy + z)\vec{k}$ is irrotational.

(b) If $\vec{r} = xi + yj + zk$, prove that $\text{div}(r^n \vec{r}) = (n+3)r^n$

OR

X. (a) Verify Stoke's theorem for $F = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.

(b) Verify divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.
