B. Tech Degree III Semester Examination, November 2007

IT/CS/EC/CE/ME/SE/EB/EI/EE 301 ENGINEERING MATHEMATICS II

(2006 Admissions)

Time : 3 Hours | Maximum Marks : 100

PART A

(Answer ***ALL*** questions)
(All questions carry ***FIVE*** marks)

(8 x 5 = 40)

I. (a) Define rank of a matrix. Find the values of l and m such that the rank of the matrix $\begin{bmatrix} 2 & 1 & -1 & 3 \\ 1 & -1 & 2 & 4 \\ 7 & -1 & l & m \end{bmatrix}$ is 2.

(b) Let T be a linear transformation from R^3 into R defined by $T(x_1, x_2, x_3) = x_1^2, x_2^2, x_3^2$. Show that T is not a linear transformation.

(c) Obtain the half range sine series for e^x in $0 < x < 1$.

(d) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.

(e) Find the inverse Laplace transform of $\log\left(\frac{s^2 + s}{s^2 + 4}\right)$.

(f) Find the Laplace transform of the saw toothed wave of period T, given $f(t) = \frac{t}{T}, 0 < t < T$.

(g) If $u = x^2 + y^2 + z^2$ and $\vec{V} = x\vec{i} + y\vec{j} + z\vec{k}$, show that div $(u\vec{v}) = 5u$.

(h) Find the work done when a force $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$ moves a particle in the xy-plane from $(0,0)$ to $(1,1)$ along the parabola $y^2 = x$.

PART B

(All questions carry ***FIFTEEN*** marks)

(4 x 15 = 60)

I. (a) Find the eigen values of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ and find the eigen vector corresponding to the largest eigen value.

(b) Find $\ker(T)$ and $ran(T)$ and their dimensions where $T : R^3 \to R^3$ defined by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + y \\ z \\ x - y \end{pmatrix}$.

OR

II. (a) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

(b) Test for consistency and solve the following system $x - y + z = 1, 2x + y - z = 2, 5x - 2y + 2z = 5$.

(Turn Over)

III. (a) Obtain the Fourier series for $f(x)=|\sin x|$ in the interval $-\pi < x < \pi$.

(b) Find the Fourier sine transform of $e^{-|x|}$. Hence show that
$$\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi}{2} e^{-m}, m > 0.$$

OR

IV. (a) Expand $f(x)$ in Fourier series in the interval $(-2,2)$ when $f(x)=\begin{cases} 0 \text{ for } -2<x<0 \\ 1 \text{ for } 0<x<2 \end{cases}$.

(b) Express the function $f(x)=\begin{cases} 1 & 0 \le x < \pi \\ 0 & x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate $\int_0^\infty \frac{(1-\cos \pi\lambda)}{\lambda} \sin x\lambda \, d\lambda$.

V. (a) Find the Laplace transforms

(i) $\frac{e^{-at}-e^{-bt}}{t}$ (ii) $\sin t\, u(t-\pi.)$

(b) Solve by the method of Laplace transforms $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$.

(c) Evaluate $\int_0^t t\, e^{-2t} \sin t \, dt$.

OR

VI. (a) Define a unit impulse function and find its Laplace transform.

(b) Apply Convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$.

(c) Find the Laplace transform of the periodic function of period $2a$ defined by
$$f(t)=\begin{cases} 1 \text{ for } 0 \le t < a \\ -1 \text{ for } a < t < 2a \end{cases}.$$

VII. (a) Prove that $div\left(\frac{\vec{r}}{r^3}\right) = 0$.

(b) Show that $\int_c \vec{F}.d\vec{r} = 3\pi$, given that $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ and C being the arc of the curve $\vec{r} = \cos t\,\vec{i} + \sin t\,\vec{j} + t\vec{k}$ from $t=0$ to $t=\pi$.

(c) If $\vec{F} = (2x^2 - 3z)\vec{i} - 2xy\,\vec{j} - 4x\vec{k}$, then evaluate $\iiint_v \nabla.\vec{F}\,dv$ where v bounded by the planes $x=0, y=0, z=0$ and $2x+2y+z=4$.

OR

VIII. (a) Find the constants a, b, c so that $\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational.

If $\vec{F} = \text{grad}\,\phi$, show that $\phi = \frac{x^2}{2} - \frac{3y^2}{2} + z^2 + 2xy + 4xz - yz$.

(b) Verify Stoke's theorem for $\vec{F} = (x^2+y^2)\vec{i} - 2xy\,\vec{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.

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