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3E1416

B. Tech. (Sem.III) (Main/Back) Examination, January - 2009
(3ME6) Advanced Engg. Mathematics (Mechanical Engg.)
(3PI6) Advanced Engg. Mathematics (Prod. & Indus. Engg.)
(3AE6) Advanced Engg. Mathematics (Automobile Engg.)

Time : 3 Hours] [Total Marks : 80
[Min. Passing Marks : 24

Attempt five questions in all. Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly.

Use of following supporting material is permitted during examination. *(Mentioned in form No. 205)*

1. Nil 2. Nil

1 (a) Find the Fourier series for $f(x) = x + x^2, \ -\pi < x < \pi$.

Hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$

(b) State and prove convolution theorem of Laplace transforms.

OR

1 (a) Find half range sine series for the function $f(x) = x$ in the interval $0 < x < 2$.

(b) Solve the integral equation

$$\int_0^\infty f(x) \cos px \, dx = e^{-p}.$$

2 (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$. It is released from rest from this position. Find the displacement

$$y(x, t) \left[wave \ eqn. \ \frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2} \right].$$

(b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2}{\partial x^2}$ by Laplace transform method, given $u(x, 0) = 3\sin 2\pi x$, $u(0, t) = 0$, $u(1, t) = 0$, where $0 < x < 1, t > 0$.

OR

2 (a) An insulated rod of length l has its ends A and B kept at $0^\circ C$ and $100^\circ C$ respectively until steady state conditions prevail. If the temperature of B is then suddenly reduced to $0^\circ C$ and kept so, while that of end A is maintained, find the temperature $u(x, t)$ at a distance x from A at time t. [Diffusion equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$].

(b) Solve by Laplace transform method $\frac{d^2 x}{dt^2} + x = t\cos 2t$ given $x(0) = x'(0) = 0$.

3 (a) If $f(z)$ is an analytic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$

(b) Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2+4)}$ at all its poles in the finite plane of z.

OR

3 (a) Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0$.

(b) Expand $\dfrac{1}{z(z-1)(z-2)}$ is Laurent's series for

(i) $|z| > 2$

(ii) $|z-1| < 1$

4 (a) Solve $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ when $u(0, y) = 0$ $u(l, y) = 0$,

$u(x, 0) = 0$ and $u(x, a) = \sin \dfrac{\pi x}{l}$.

(b) Show that

$$J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x).$$

OR

4 (a) Show that

(i) $\dfrac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$, $n \geq 0$

(ii) $\dfrac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}(x)$, $n \geq 0$

(b) Show that

$$P_n(x) = \frac{1}{\lfloor n \, 2^n} \frac{d^n}{dx^n} (x^2 - 1)^n.$$

Hence find $P_5(x)$.

5 (a) Prove that

$$u_0 + \frac{x u_1}{\lfloor 1} + \frac{x^2 u_2}{\lfloor 2} + \frac{x^3 u_3}{\lfloor 3} + \ldots$$

$$= e^x \left[u_0 + x \, \Delta u_0 + \frac{x^2}{\lfloor 2} \Delta^2 u_0 + \ldots \right]$$

(b) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by (i) Simpson's $\frac{1}{3}$ rule and (ii) Trapezoidal rule. Hence obtain the value of π by result obtained from (i) [Take six intervals].

OR

5 (a) From the following table find $y(0.25)$, $y(0.62)$ and $y(0.43)$

x	0.0	0.2	0.4	0.6	0.8
y	0.3989	0.3910	0.3683	0.3332	0.2897

(b) Using Lagrange's interpolation formula, find y for $x = 10$ from the following table :

x	5	6	9	11
y	12	13	14	16