

Total number of printed pages – 6

MCA  
SCM 2005

First Semester Examination – 2008

DISCRETE MATHEMATICS

Full Marks – 70

Time – 3 Hours

Answer Question No. 1 which is compulsory and any five from the rest.

The figures in the right hand margin indicate full marks for the questions.

1. Answer the following questions : 2×10

(i) Is the following statement a tautology ?  
Justify your answer.

$$p \Rightarrow (q \Rightarrow p)$$

(ii) Let  $A = \{a, b, c, d\}$ . The relation  $R = \{(a, a), (a, b), (b, c), (c, d)\}$  is defined on  $A$ . Find  $R^2$ .

P.T.O.

(iii) If  $\{(1, 3, 5), (2, 4)\}$  is a partition of the set  $A = \{1, 2, 3, 4, 5\}$ , determine the corresponding equivalence relation  $R$ .

(iv) What do you mean by symmetric closure of a relation? How can you find the symmetric closure of a relation  $R$  defined on the set  $A$ ?

(v) Can you find a cycle of length greater than one in a diagraph representing a partial order relation? Justify your answer.

(vi) For any two elements  $a, b$  of a lattice, show that  $a \vee b = b$  if  $a \leq b$ .

(vii) Is there any Boolean algebra having nine elements? Justify your answer.

(viii) Write the algorithm of preorder and post order searching of a tree.

(ix) How can you know whether an undirected graph possesses an Euler path or not?

(x) What do you mean by minimum distance of an encoding function? How can you find the minimum distance of a group code?

2. (a) Prove by mathematical induction that  $n$  divides  $n^3 - n$  for every positive integer  $n$ .

5

(b) Find an explicit formula for the sequence defined by  $C_n = 3C_{n-1} - 2C_{n-2}$  with initial conditions  $C_1 = 5$  and  $C_2 = 3$ .

5

3. (a) If  $R$  is a relation defined on  $A = \{a_1, a_2, a_3, \dots, a_n\}$ , then show that  $M_{R^2} = M_R \otimes M_R$ .

5

(b) State Warshall Algorithm to find the transitive closure of a relation. Hence, find the transitive closure of a relation  $R$  defined on the set  $\{a, b, c, d\}$  whose matrix representation is given below.

5

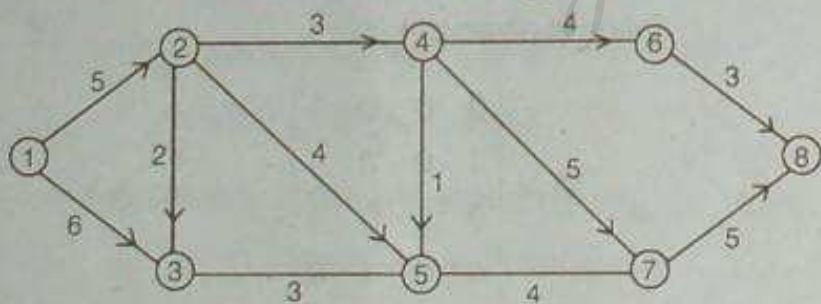
$$M_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

4. (a) Let  $L$  be a bounded lattice with at least two elements. Show that no element of  $L$  is its own complement. 5

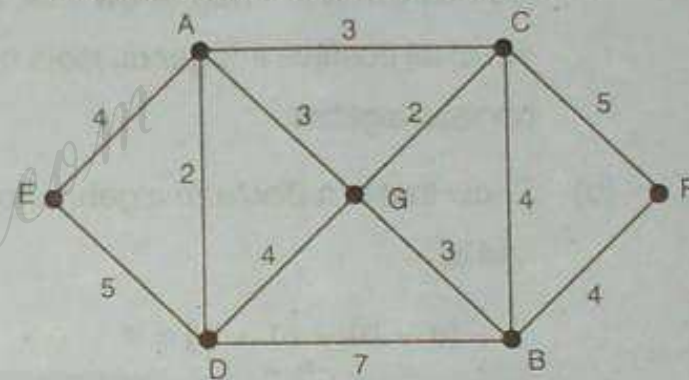
(b) Let  $A$  be a finite nonempty poset with partial order  $\leq$ . Show that  $A$  has at least one maximal element and at least one minimal element. 5

5. (a) Let the number of edges of a graph  $G$  be  $m$  and number of vertices be  $n$ . Then show that  $G$  has a Hamiltonian circuit if  $m \geq \frac{1}{2}(n^2 - 3n + 6)$ . 5

(b) Find a maximum flow in the given network where each edge is labelled with its maximum capacity; node 1 being the source and node 8 being the sink. 5



6. (a) Write the Kruskal's algorithm to find a minimal spanning tree for a graph. Using the algorithm, find a minimal spanning tree for the following graph. List the edges in the order in which they are chosen. 5



(b) Let  $R$  be a symmetric relation on a set  $A$ . Then prove that the following statements are equivalent. 5

- (i)  $R$  is an undirected tree
- (ii)  $R$  is connected and acyclic.

7. (a) Show that the intersection of two congruence relation on a semigroup is a congruence relation. 5

(b) Let  $G$  be an abelian group with identity element  $e$  and  $H = \{x : x^2 = e\}$ . Show that  $H$  is a sub group of  $G$ . 5

8. (a) Let  $n = p_1 p_2 p_3 \dots p_k$ , where the  $p_i$  are distinct primes. Then show that  $D_n$ , the set of all positive integer divisors of  $n$ , is a boolean algebra. 5

(b) Show that in a Boolean algebra, for any  $a$  and  $b$ ,

$$(a \vee b) \vee (a \wedge b) = a \quad 5$$

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