Total number of printed pages – 7

B. Tech

BSCM 3301

Fifth Semester Examination – 2007

DISCRETE MATHEMATICAL STRUCTURES

Full Marks – 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2 × 10

(i) Define scope of a quantifier and give an example.

(ii) What is Euclidian algorithm to find the GCD of two integers ?

P.T.O.

(iii) What is lexicographic ordering on N X N, where N is the set of natural numbers ?

(iv) Give a recursive definition of the set of all positive integers congruent to 2 modulo 3.

(v) How many positive integers less than 1000 exit which are divisible by 7 but not by 11 ?

(vi) What is symmetric closure of the relation $R = \{ (a, b) \mid a > b \}$?

(vii) What are the principles of drawing Hasse diagram of a partial ordering relation ?

(viii) Define chromatic number of a simple graph and that of a planar graph.

(ix) If $(\{a, b\}, *)$ be a semigroup where $a * a = b$ then can you say that $b * b = b$? Justify your answer.

(x) Let a, b are two elements of a lattice $(A, \leq)$. Show that $a \wedge b = b$ iff $a \vee b = a$.

2. (a) Show that the hypotheses $(p \wedge q) \vee r$ and $r \rightarrow s$ imply the inclusion $p \vee s$. 5

(b) Using mathematical induction show that $n^3 - n$ is divisible by 3 whenever n is a positive integer. 5

3. (a) Using Warshall's algorithm find the transitive closure of the relation whose matrix

representation is given below. 5

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

(b) Let R be a relation on the set of ordered pairs of positive integers such that

$((a, b), (c, d)) \in R$ if and only if $ad = bc$

Show that R is an equivalence relation. 5

4. (a) Show that the relation 'set inclusion' on the power set of S is a partial ordering relation. Draw the Hasse diagram of the relation where S ={a , b, c}. 5

(b) How do you differentiate between the maximal element and greatest element of a

poset ? Show that there is exactly one greatest element of a poset, if such an element exits. What about maximal element ? 5

5. (a) Solve the following recursive relation

$a_r = a_{r-1} - a_{r-2}$, given that $a_1 = 1$ and $a_2 = 0$ 5

(b) Use generating function to find the number of ways to select r objects of n different kinds if we must select at least one object of each kind. 5

6. (a) Define a tree. Show that an undirected graph is a tree if and only if there is an unique simple path between any two of its vertices. 5

(b) Show that a connected multigraph with at least two vertices has an Euler circuit if and only if each of its vertices has even degree. 5

7\. (a) Let (A, *) be a monoid such that for every $x \in A$, $x * x = e$, where e is the identity element. Show that (A, *) is an abelian group. 5

(b) Let (A, $\leq$) be a distributive lattice. If $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some a, then show that $x = y$. 5

8\. (a) Explain the principle of inclusion- exclusion. Using this principle, find out the number of solutions for the equation $x_1 + x_2 + x_3 = 13$, where x_1, x_2 and x_3 are non-negative integers less than six. 5

(b) Find the total number of strings of 10 ternary digits (0, 1 or 2) which contain exactly two 0s, three 1s and five 2s. 5

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