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B. Tech

BSCM 3301

Fifth Semester Examination – 2008

DISCRETE MATHEMATICAL STRUCTURES

Full Marks – 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

   (i) Express the following system specification using predicate, quantifier and logical connectives. "At least one mail message, among the non-empty set of messages, can be saved if there is a disk with more than 10 kilobytes of free space."

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(ii) Define a countable set. Give an example of a countable set.

(iii) Draw the Hasse diagram of the partial order relation "greater than equal to" on the set {1, 2, 3, 4, 5}.

(iv) In how many ways three jobs can be assigned to five employees if each employee can be given more than one job ?

(v) How many positive integers less than 1000 exist which are divisible by 5 but not by 7 ?

(vi) What is the transitive closure of the relation R = { (a, b), (b, a), (c, d), (d, c)} ?

(vii) Let S = {1, 2, 3, 4, 5, 6}. The collection of sets A ={1, 2, 3}, B ={4, 5} and C= {6} forms a partition of the set S. Find the equivalence relation on S corresponding to this partition.

(viii) Define chromatic number. What is the chromatic number corresponding to a polygon of 10 sides ?

(ix) The order of a group is prime. Can you find a subgroup other than the trivial subgroups ? Justify.

(x) Find the inverse of each element, if exist, of the lattice D_{30}, where a ≤ b if a divides b and D_{30} represents the set of all divisors of 30.

2. (a) Verify that the following program segment is correct with respect to the initial assertion y = 3 and the final assertion z = 6

```
x : = 2
z : = x + y
if y > 0 then
    z : = z + 1
else
    z : = 0
```

5

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(b) Using mathematical induction show that for any positive integer n,

$3 . + 3 . 5 + 3 . 5^2 + 3 . 5^3 + \ldots + 3 . 5^n = 3(5^{n+1} - 1)/4$ 5

3. (a) Find the number of ways in which 25 identical cookies be distributed among four children so that each child gets at least three but no more than seven cookies. 5

(b) Use generating function to solve the recurrence relation $a_k = 3a_{k-1} + 4^{k-1}$ with the initial condition $a_0 = 1$. 5

4. (a) Using Warshall's algorithm find the transitive closure of the relation whose matrix representation is given below. 5

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) Define closure of a relation with respect to a property P. Suppose that the relation R on the finite set A is represented by the matrix M_R. Show that the matrix that represents the reflexive closure of R is $M_R \vee I_n$. 5

5. (a) Prove that a connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree. 5

(b) Show that a simple graph G is bipartite if only if it has no circuit with odd number of edges. 5

6. (a) Show that a simple graph that has a circuit with odd number of vertices in it can not be colored using two colors. 5

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(b) Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion; if a player is eliminated after one loss and games are played until one entrant has not lost. 5

7. (a) Let (A, *) be a semigroup and e be a left identity. Furthermore, for every x in A there exist y in A such that y * x = e. Then

(i) show that for every a, b, c in A, if a * b = a * c then b = c

(ii) show that (a , *) is a group by showing that e is an identity element. 5

(b) Let (H ,*) be a subgroup of (G ,*). Let $N = \{x : x \varepsilon G, xHx^{-1} = H\}$. Then show that (N ,*) is a subgroup of (G ,*). 5

8. (a) Let $E(x_1, x_2, x_3) = \overline{(\overline{x_1 \vee x_2}) \vee (\overline{x_1} \wedge x_3)}$ be a boolean expression over two-valued boolean algebra. Write $E(x_1, x_2, x_3)$ both in disjunctive and conjunctive normal forms. 5

(b) For any a and b in a lattice, show that

(i) $a \vee (a \wedge b) = a$

(ii) $a \wedge (a \vee b) = a$ 5

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