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2235 A

M.A. Winter Semester

ECONOMICS

Course 104 – Game Theory - II

(Admissions of 1999 & onwards)

Time : 2-1/2 hours Maximum Marks : 70

(Write your Roll No. on the top of immediately on receipt of this question paper).

Attempt as many as you want.

1.

a) Define a TU game and an allocation of a TU game. [3]

b) Define Symmetry, Dummy and Additivity in this context. [3]

c) Define the Shapley value. [3]

d) Show that the Shapley value satisfies the above properties. [5]

e) Propose an allocation which satisfies Symmetry and Additivity but violates Dummy. [3]

f) Define a convex game. [3]

g) A game (N, v) is called superadditive if for all S, T such that $S \cap T = \emptyset$, $v(S \cup T) \geq v(s) + v(T)$. Show that convex games are also superadditive. [5]

h) Give an example of a superadditive game which is not convex. [5]

2. Consider an auction setting. Suppose there are just two bidders and their valuations are independently drawn from a uniform distribution on $[0, 1]$. Consider a second price auction.

a) Show that truth telling is a dominant strategy? [2]

b) Show that truth telling is a Bayes-Nash equilibrium? [4]

c) Find the seller's expected revenue. [4]

d) Define a second price auction with a reserve price. [2]

e) Show that truth telling is still a Bayes-Nash equilibrium? [2]

f) Find the seller's expected revenue. [4]

g) Can you use the 'revenue equivalence principle' to compare (*c*) and (*f*)? Why or why not? [2]

3. Consider a bilateral trade setting in which buyer's and seller's valuations (θ_B and θ_S respectively) of an indivisible object are drawn independently from the uniform distribution on $[0, 1]$. Utility of buyer and seller are as follows, $u_i = \theta_i.\mathbf{1}_i +$ transfer to i, where $\mathbf{1}_i = 1$ if i gets the object and 0 otherwise. Suppose that θ_i is private information of agent i.

a) Model this situation as a mechanism design problem (specify the Bayesian game) [3]

b) Define efficieny, individual rationality and budget balanced-ness in this context? [3]

b) Find the 'pivotal' mechanism. [3]

c) Is 'pivotal' mechanism individually rational? [3]

d) Find a mechanism which is Bayesian incentive compatible (where truth telling is Bayesian-Nash equilibrium), efficient and individually rational. [5]

e) Show that if a mechanism is Bayesian incentive compatible, efficient and individually rational then the sum of the buyer's and seller's expected utilities can not be less than $\frac{5}{6}$. [5]

f) Show that if a mechanism is budget balanced then the sum of the buyer's and seller's expected utilities can not exceed $\frac{2}{3}$. [5]

g) Show that there does not exist any Bayesian incentive compatible, efficient, individually rational and budget balanced mechanism in bilateral trading. [3]

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