This question paper contains 3 printed pages.

2253

Your Roll No. ....

M.A. / Winter Semester **A**

ECONOMICS

Course 701— Population and Development

(Admissions of 1999 and onwards)

Time : 21/2 hours *Maximum Marks* : 70

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory. Please be brief in your answers or marks would be deducted for verbosity.

Question 1. Consider a game with two players, 1 and 2. Suppose both players have the same strategy set A. Let $u_i : A^2 \to \Re$ be player i's payoff function. Suppose

(a) A is a nonempty, compact and convex subset of $\Re^l$, where $l \in \mathcal{N}$,

(b) for all $a, b \in A$, $u_1(a,b) = u_2(b,a)$,

(c) for every $a \in A$, $u_1(.,a) : A \to \Re$ is quasi-concave, and

(d) u_1 is continuous.

Show that:

(A) u_2 is continuous and $u_2(a,.) : A \to \Re$ is quasi-concave for every $a \in A$.

(B) diag $A^2 = \{(a,b) \in A^2 \mid a = b\}$ is nonempty, convex and compact.

(C) Provide a fixed point argument on diag A^2 to show the existence of $a^* \in A$ such that $u_1(a^*,a^*) \geq u_1(b,a^*)$ and $u_2(a^*,a^*) \geq u_2(a^*,b)$ for every $b \in A$.

(D) Provide an alternative simpler proof of (C) using a fixed point argument on A.

$(3, 5, 9, 6\frac{1}{3})$

Question 2. Consider an open set $C \subset \Re$ and $f : C \to \Re$. f is said to locally bounded if, for every $x \in C$, there exists $r > 0$ and $M \in \Re_+$, such that $|f(y)| \leq M$ for every $y \in B_r(x)$.

(A) Suppose C is convex and f is convex. Show that f is locally bounded.

(Hint: Find $r > 0$ such that $B_r(x) \subset C$. Consider $y \in B_r(x)$. Use the convexity of f to get an upper bound on $f(y)$ that is independent of y, say M. Noting that there exists

Turn over

$z \in B_r(x)$ such that $x = y/2 + z/2$, use the convexity of f to get a lower bound on $f(y)$ that is independent of y.)

(B) By (A), there exists $r > 0$ and $M \in \Re_+$ such that $B_{2r}(x) \subset C$ and $|f(y)| \leq M$ for every $y \in B_{2r}(x)$. Consider distinct $y, z \in B_r(x)$ and set $w = z + (r/\alpha)(z - y)$, where $\alpha = |y - z|$. Show that $w \in B_{2r}(x)$ and that z is a convex combination of y and w.

(C) Use the convexity of f to show that

$$f(z) - f(y) \leq \frac{\alpha}{r}|f(w) - f(y)| \leq \frac{2M}{r}|z - y|$$

(D) Show that $|f(z) - f(y)| \leq (2M/r)|z - y|$, and therefore f is continuous.

(E) Provide *two* economic applications of this result.

$(5, 3, 8\frac{1}{3}, 3, 4)$

Question 3. Consider the pair $(X, \succeq)$, where $\succeq$ is a binary relation on the set X. Let $x \sim y$ if and only if $x \succeq y$ and $y \succeq x$. Let $x \succ y$ if and only if $x \succeq y$ and $\neg y \succeq x$. Suppose

(a) $\succeq$ is a complete preordering on X;

(b) there exist a, $b \in X$ such that $b \succeq x$ and $x \succeq a$ for every $x \in X$;

(c) X is convex;

(d) for every $x \in X$, the sets $U^x = \{t \in [0, 1] \mid tb + (1 - t)a \succeq x\}$ and $U_x = \{t \in [0, 1] \mid x \succeq tb + (1 - t)a\}$ are closed in $[0, 1]$;

(e) for all s, $t \in [0, 1]$, $s > t$ if and only if $sb + (1 - s)a \succ tb + (1 - t)a$;

(f) for all x, y, $z \in X$ and $t \in [0, 1]$, if $x \sim y$, then $tx + (1 - t)z \sim ty + (1 - t)z$.

Given the above assumptions, show that

(A) there exists $f : X \to \Re$ such that $x \succ y$ if and only if $f(x) > f(y)$, and $f(tx + (1 - t)y) = tf(x) + (1 - t)f(y)$ for all $x, y \in X$ and $t \in [0, 1]$.

(Hint: Show that, for every $x \in X$, there exists $t \in [0, 1]$ such that $x \sim tb + (1 - t)a$.)

(B) Provide an economic application of (A).

$(16, 7\frac{1}{3})$

Question 4. Suppose there are two dates, 0 and 1. Suppose the world will be in one of p states at date 1, but the true state of the world at date 1 is unknown at date 0. Let there be n financial assets. Let A be a $p \times n$ matrix where a_{ij} is interpreted as the payment by asset j in state i at date 1; assume that $a^i = (a_{i1}, \ldots, a_{in}) \neq 0$ for $i = 1, \ldots, p$. Let $b \in \Re^n - \{0\}$ be the vector of financial asset prices at date 0, with b_j as the price of asset j.

(A) Provide an economic interpretation of the condition: for every $x \in \Re^n$,

$$Ax \geq 0 \qquad \Rightarrow \qquad \langle b, x \rangle \geq 0 \tag{$*$}$$

(B) Show that, if $(*)$ is satisfies, then there exists $c \in \Re^p_+ - \{0\}$ such that $b = cA$.

(C) Provide economic interpretations of (B).

$(2, 16\frac{1}{3}, 5)$