[This question paper contains 7 printed pages]

Your Roll No

7017 **J**

M.Tech/II Sem

NUCLEAR SCIENCE & TECHNOLOGY

Paper NST – 608 Mathematical & Numerical Methods in Nuclear Engineering

Time 3 Hours *Maximum Marks* 70

(*Write your Roll No on the top immediately on receipt of this question paper*)

Attempt **all** *questions*

1 Attempt ***any five*** of the following Each part carries ***two*** marks

(a) How many real zeroes does the function $f(x) = 2x^3 + 2x^2 + 5x + k$ has ? (k is any real number)

(b) Ten iterations of the bisection method are applied to the function

$$f(x) = 2x^3 + x^2 - 2x - 6$$

to find its zero lying between 1 and 2 To how many decimal places the result is expected to be correct?

(c) Given that $f(x)$ = 0 479, 0 565, 0 644 at x = 0 5, 0 6 and 0 7 respectively, use the three-point difference formulas to find $f'(x)$ at x = 0 5, 0 6 and 0 7

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(d) The integral $\int_0^1 e^x \, dx$ is obtained by composite Simpson rule. If the error is to be less than 10^{-8}, find the minimum number of subintervals required.

(e) For the linear system

$$\begin{aligned} x_1 + x_2 + x_3 &= 4, \\ x_1 + 2\alpha x_2 + x_3 &= 6, \\ \alpha x_1 + x_2 + (2-\alpha)x_3 &= 4, \end{aligned}$$

find α for which the system has (i) no solution, (ii) infinite number of solutions.

(f) For what values of k is the matrix

$$A = \begin{pmatrix} 0 & k \\ k & 0 \end{pmatrix}$$

convergent ?

(g) Find the spectral radius, the l_2 norm and the l_∞ norm of the matrix

$$A = \begin{pmatrix} 2 & 2 \\ 3 & [illegible] \end{pmatrix}$$

(h) Show that the initial value problem

$$y'(t) = ty^{[illegible]} + e^{t}, 0 \le t \le 1, y(0) = 1,$$

has a unique solution by applying the relevant theorem. 2×5=10

2. Attempt ***any five*** of the followings. Each part carries ***four*** marks.

(a) Show that the function

$$f(x) = \frac{x^4 - 3x^2 + 3}{x^4 + 2}$$

has a fixed point in $x \in [0, 2]$. Is the fixed point unique ?

(b) Find c_0, c_1 and x_1, so that the quadrature formula

$$\int_0^1 f(x)\,dx = c_0 f(0) + c_1 f(x_1)$$

has the highest degree of precision.

(c) Show that the inverse of a non-singular lower triangular matrix is a lower triangular matrix.

(d) Show that the matrix AB is non-singular, if and only if, both A and B are non-singular.

(e) Show that for any vector $x \in \mathbb{R}^n$,

$$\|x\|_1 \geq \|x\|_2 \geq \|x\|_\infty$$

where

$$\|\ \|_1 = \sum_{i=1}^{n} |x_i|, \quad \|\ \|_2 = \sqrt{\sum_{i=1}^{r} x_i^2}, \quad \|\ \|_\infty = \max_{1 \leq i < n} |x_i|$$

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(f) The initial value problem

$$y' = \frac{y}{t} - \left(\frac{y}{t}\right)^2, \quad 1 \le t \le 2, y(1) = 1$$

has the solution $y(t) = \frac{t}{1 + \ln(t)}$. Find the error bound in the value of $y(2)$ obtained by Euler's method with $h = 0.1$

(g) Show that for the initial value problem

$$y''(x) = p(x)\, y'(x) + q(x)\, y(x),$$

$$a \le x \le b, \quad y(a) = 0, y'(a) = 1,$$

if $q(x)$ and $p(x)$ are continuous an $[a, b]$ and $q(x) > 0$ on $[a, b]$, then $y(b)$ cannot be zero

(h) The partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y), u(x, y) = g(x, y)$$

on the boundary of $0 < x < 1$, $0 < y < 1$, is to be solved by finite difference method

$$u(x_{i+1}, y_j) - 2u(x_i, y_j) + u(x_{i-1}, y_j)$$

$$+ h^2 / k^2 [u(x_i, y_{j+1}) - 2u(x_i, y_j) + 4(x_i, y_{j-1})]$$

$$= h^2 f(x_i, y_j)$$

by dividing both the intervals (0, 1) into three equal parts, write down the equations obtained in a convenient form 4×5=20

3 Attempt ***any five*** of the following. Each part carries 8 marks

(a) Obtain the formula for finding the zeroes of a function by Newton's method Interpret it geometrically How is this method modified to obviate the need for evaluation of the derivative ?

(b) Show that for the Simpson rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 4f(x_1) + f(b)\right],$$

$$h = \frac{b-a}{2},\ x_1 = \frac{b+a}{2},$$ the error term is

$$-\frac{h^5}{90} f^{(4)}(\xi)$$ for source $\xi \in [a, b]$

(c) Show that for any polynomial $P(x)$ of degree less than $2n$,

$$\int_{-1}^{1} P(x)dx = \sum_{j=1}^{n} e_j P(x_j),$$

where x_i are the zeroes of Legendre polynomials $P_n(x)$ and e_i are given by

$$c_i = \int_{-1}^{1} \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j} \, dx$$

(d) Show that for any vector $x^{(0)} \in \mathbb{R}^n$, the sequence $\left\{x^{(k)}\right\}_{k=0}^{\infty}$ defined by

$$x^{(k)} = \mathrm{T}^{(k-1)} + \mathrm{C} \;\forall\; k \geq 1,$$

converges to the unique solution of

$$x = \mathrm{T}x + \mathrm{C}$$

(where T is an $n \times n$ matrix, and C a vector) if and only if $\rho(\mathrm{T})$, the spectral radius of T is less than 1 Reduce the Jacobi and Gauss-Seidel techniques for solving the linear system $\mathrm{A}x = b$ to the above form and thereby state the corresponding result for these techniques

(e) Describe the Q-R algorithm for finding the eigen-values of a tridiagonal symmetric matrix

(f) What is the main drawback of the Taylor's methods for solving an initial value problem Describe the basic idea of the Runge-Kutta class of methods for such problems Obtain the formula for the mid-point method, a Runge-Kutta method of order 2

(g) Describe the finite difference method for the solution of the boundary value problem

$$y''(x) = p(x)\,y'(x) + q(x)\,y(x) + r(x),$$

$$a \le x \le b, \quad y(a) = \alpha, \quad y(b) = \beta$$

What are the conditions that need to be satisfied for the method to work ?

(h) Describe the finite difference method for the solution of parabolic type of partial differential equation

$$\frac{\partial u}{\partial t}(x,t) = \alpha^2 \frac{\partial^2 u}{\partial x^2}(x,t),\ 0 < x < l,\ t > 0,$$

$$u(0,t) = u(l,t) = 0,\ t > 0, \quad u(x,0) = f(x),\ 0 \le x \le l$$

What is the drawback of the forward difference method and how is it corrected in the backward difference method ? 8×5=40