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Your Roll No .

5187

B.Sc. (Prog.) / II J

MA-202 : MATHEMATICS – II – Algebra & Differential Equations

(For Physical Sciences/Applied Physical Sciences)

(Admissions of 2008 and onwards)

Time : 3 Hours **Maximum Marks : 112**

(*Write your Roll No on the top immediately on receipt of this question paper*)

Attempt **two** parts from each question

All questions are compulsory

UNIT – I

1\. (a) Let $G = \{(a, b) \mid a, b \in \mathbb{R}, b \neq 0\}$ Define a binary operation '0' on G by (a, b) 0 (c, d) = (a + bc, bd) Show that (G, 0) is a non-abelian group. $7\frac{1}{2}$

(b) Let H be a subgroup of G. Show that C(H) = $\{x \in G \mid xh = hx \text{ for all } h \in H\}$ is a subgroup of G $7\frac{1}{2}$

(c) Is U(8) cyclic group ? Justify your answer $7\frac{1}{2}$

2. (a) If H is a subgroup of a cyclic group, show that H is also cyclic $7\frac{1}{2}$

(b) If H is a subgroup of G, show that $Ha \cap Hb = \phi$ or $Ha = Hb$. $7\frac{1}{2}$

(c) Let G be a group. Let $a, b \in G$ If $ab = ba$ and g.c d. $(0(a), 0(b)) = 1$, show that $0(ab) = 0(a)\,0(b)$. $7\frac{1}{2}$

3. (a) Let H be a subgroup of G such that index of H in G is 2. Show that H is normal in G $7\frac{1}{2}$

(b) Let N be a normal subgroup of G such that $0\left(\frac{G}{N}\right) = m$ and H is a subgroup of G such that $0(H) = n$ and g c d $(m, n) = 1$, then show that $H \subseteq N$ $7\frac{1}{2}$

(c) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & -6 \end{pmatrix}$

$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{pmatrix}$

Compute $\beta^{-1}\,\alpha\,\beta$ and find its order. $7\frac{1}{2}$

UNIT – II

4 (a) Solve

(i) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$,

(ii) $p^3(x + 2y) + 3p^2(x + y) + (y + 2x)\,p = 0$, $p = \frac{dy}{dx}$ 6, $5\frac{1}{2}$

(b) Solve by the method of variation of parameters :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$$ $11\frac{1}{2}$

(c) Prove that the Wronskian of two solutions of the second order homogenous linear differential equation

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$$

where a_0, a_1, a_2 are continuous real valued functions of x defined on (a, b) and $a_0(x) \neq 0$ for any x in (a, b), is either identically zero or never zero on (a, b). $11\frac{1}{2}$

5 (a) Solve :

$$\frac{dx}{dt} + 2\frac{dy}{dt} - 2x + 2y = 3e^t,$$

$$3\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 4e^{2t},$$ 11

(b) Solve .

$$yz(1 + 4xz)dx - xz(1 + 2xz)dy - xy\,dz = 0.$$ 11

(c) An 8 lb weight is attached to the lower end of a coil spring suspended from a fixed support The weight comes to rest in its equilibrium position, thereby stretching the spring 6 in The weight is then pulled down 9 in below its equilibrium position and released at t = 0. The medium offers a resistance in pounds numerically equal to $4\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instantaneous velocity in feet per second Determine the displacement of the weight as a function of the time 11

UNIT – III

6 (a) Find the general integral of
$(z^2 - 2yz - y^2)p + (xy + xz)q = xy - xz,$
where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ 11

(b) Find the complete integral of
$z^2 = pq\,xy$
where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ 11

(c) Reduce the equation
$\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$
to the canonical form 11