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Your Roll No...........................

6360

B.Sc. (Hons.) III Sem./II Yr./NS **H**

COMPUTER SCIENCE

Paper 303 : Algebra

(Admissions of 2001 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer *All* the questions. Use of scientific calculator is allowed. Parts of a question must be answered together. *All* questions are of equal marks.

1. Show that the set :

$$G = \left\{x + y\sqrt{3} : x, y \in Q\right\}$$

is a group with respect to addition.

2. Define the centre of a group. Show that the centre of a group G is a sub-group of G. Find the centre of S_3.

P.T.O.

3. Show that the mapping $f : C \to C$ of complex conjugation is an isomorphism of rings with respect to usual addition and multiplication.

4. Show that the map $F : Z[X] \to Q$ defined by $F(f) = f(1/2)$ is a morphism of rings and determine its kernel and its image.

5. Find the 10th power of the matrix :

$$A = \begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix},$$

by finding suitable matrices P and D such that $A = PDP^{-1}$.

6. Draw the Hasse diagram representing the partial ordering $\{(a, b) : a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 8, 12\}$. Identify the maximal and minimal elements.

7. Let S be a convex set in a vector space V. Let w be an arbitrary element of V. Show that the translation map $T_w(S)$ is convex.

8. Find the coordinates of the vector X with respect to the vectors A, B and C where $X = (0, 0, 1)$; $A = (1, 1, 1)$; $B = (-1, 1, 0)$; $C = (1, 0, -1)$.

9. Let F : V $\to$ W be a linear map, show that the image of F is a subspace of W and kernel of F is a subspace of V.

10. Let V be the vector space generated by the three functions :

$$f_1(t) = 1;\ f_2(t) = t;\ f_3(t) = t^2.$$

Let D : V $\to$ V be the derivative. What is the matrix of D with respect to the basis $\{f_1, f_2, f_3\}$?

11. State and prove Schwarz inequality. Let V be the space of continuous functions on $[0, 2\pi]$, and let the scalar product be given by :

$$\langle f, g \rangle = \int_0^{2\pi} f(x)\, g(x)\, dx .$$

If $f(x) = x^2$ then find $\|f\|$.

12. Prove that a mapping F : V $\to$ W has an inverse if and only if it is both injective and surjective. Hence show that the linear map L : $R^2 \to R^2$ defined by $L(x, y) = (2x + y, 3x - 5y)$ is invertible.

13. Let P be a parallelogram spanned by the vectors $(-1, 2)$ and $(2, 0)$. Let $L : R^2 \to R^2$ be a map defined by $L(x, y) = (x - 2y, x + y)$. Find the area of $L(P)$.

14. Find the maximum and minimum of the function $2x^2 - 4xy + 3y^2$ on the unit circle.

15. Find the orthonormal basis for the subspace U of R^4 spanned by $v_1 = (1, 1, 1, 1)$; $v_2 = (1, 2, 4, 5)$; $v_3 = (1, -3, -4, -2)$.