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Your Roll No................

1977

B.Sc. (Hons.) II Sem./NS **G**

COMPUTER SCIENCE

Paper 203—Calculus II

(New Course)

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks :* 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. (*a*) Use the max-min inequality to show that if f is integrable then :

$$f(x) \geq 0 \text{ on } [a, b] \Rightarrow \int_a^b f(x)\,dx \geq 0$$

and

$$f(x) \leq 0 \text{ on } [a, b] \Rightarrow \int_a^b f(x)\,dx \leq 0.$$ 5

P.T.O.

(b) Show that if f is continuous on $[a, b]$ $a \neq b$ and if

$$\int_a^b f(x)\, dx = 0$$

then $f(x) = 0$ at least once in $[a, b]$. 5

2. (a) The velocity of a particle moving in space is

$$\frac{d\vec{r}}{dt} = (t^3 + 4t)\,\hat{i} + t\hat{j} + 2t^2\hat{k}\cdot$$

Find the particles position as a fn. of t if

$\vec{r} = \hat{i} + \hat{j}$ when $t = 0$. 5

(b) The region bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$ is revolved about the x-axis to generate a solid. Using Shell's formula find the volume of the solid. 5

3. (a) Find the volume of a solid using slicing method when the solid lies between planes perpendicular to x-axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the axis between these planes are

vertical squares whose base edges run from the semicircle $y = -\sqrt{1 - x^2}$ to the semicircle $y = \sqrt{1 - x^2}$. 5

(b) Find the area of the surface generated by revolving the curve $y = x^3$, $0 \le x \le \frac{1}{2}$, about the x-axis. 5

4. (a) Find the centre of mass of a thin plate of density $\delta = 3$ bounded by the lines $x = 0$, $y = x$ and the parabola $y = 2 - x^2$ in the first quadrant. 5

(b) Evaluate :

$$\iint_R e^{x^2 + y^2} \, dydx$$

where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1 - x^2}$. 5

5. (a) Let D be the region in xyz-space defined by the inequalities :

$$1 \le x \le 2, \; 0 \le xy \le 2, \; 0 \le z \le 1.$$

P.T.O.

Evaluate :

$$\iiint_D (x^2y + 3xyz)\, dx\, dy\, dz$$

by applying the transformations :

$$u = x,\ v = xy \text{ and } w = 3z$$

and integrating over the appropriate region G in UVW plane. 5

(b) Find an analytic function whose real part is given function $U(x, y) = x - xy$. 5

6. (a) Find the images of x = constant and y = constant under $f(z) = \sin z$. 5

(b) State Cauchy integral theorem. Use it to find the value of $\int_C \frac{z+4}{z^2+2z+5}\, dz$, if c is the circle $|z + 1| = 1$. 5

(c) Use Residue theorem to evaluate :

$$\int_0^{2\pi} \frac{d\theta}{5 - 4\sin\theta}.$$ 5

7. (a) Show that the Fourier series :

$$\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^{n-1}\left[\frac{\sin nx}{2n} - \frac{\cos nx}{n^2}\right]$$

converges to the periodic fn. f in $]-\pi, \pi[$

where

$$f(x) = \begin{cases} x^2 + x & \text{for } -\pi < x < \pi \\ \pi^2 & \text{for } x = \pm\pi \end{cases}$$ 5

(b) Expand in a series of sines and consines of multiple angles of x, the periodic fn. f with period 2π defined as :

$$f(x) = \begin{cases} -1 & \text{for } -\pi \le x < 0 \\ 1 & \text{for } 0 \le x \le \pi \end{cases}$$

Also calculate the sum of the series at $x = \frac{\pi}{2}$. 5