Cat - A

This question paper contains 7 printed pages.]

933 *Your Roll No.* ....

2009

B.A. Prog./I **I**

(A)

MATHEMATICS

Paper I

(Algebra and Calculus)

(New Course : Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note : *The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.*

All sections are compulsory and have equal marks.

[P.T.O.

Section I

1. (a) Find non-singular matrices P and Q such that PAQ is in the normal form where

$$A = \begin{pmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{pmatrix}$$

(b) Show that the set $S = \{x, y, 2x - 3y\}$ x, y are real numbers} is a subspace of $\mathbb{R}^3$.

Or

(a) Using Cayley-Hamilton theorem determine the inverse of the matrix

$$\begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$$

(b) Solve the following system of linear equations :

$$x - 3y + 2z = 0$$
$$7x - 21y + 14z = 0$$
$$-3x + 9y - 6z = 0$$

Section II

2. (a) Express $\sin^5\theta \cos^2\theta$ in a series of sines of multiples of θ.

(b) If $\alpha, \beta, \gamma, \delta$ are the roots of the equations

$x^4 + px^3 + qx^2 + rx + s = 0, s \neq 0$

find the values of

(i) $\sum \alpha^2$ (ii) $\sum \alpha^2\beta^2$

Or

(a) Solve the equation $x^4 + 4x^3 + 6x^2 + 4x + 5 = 0$, whose one root is i.

(b) Prove that

$1 + \cos 9\theta = (1 + \cos\theta)[16\cos^4\theta - 8\cos^3\theta - 12\cos^2\theta + 4\cos\theta + 1]^2$.

Section III

3. (a) Examine the function

$$f(x) = \begin{cases} x^m \sin\frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

for derivability at the origin. Also determine m when $f'(x)$ is continuous at origin.

(b) If $V = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} - \frac{1}{2}\tan V = 0.$$

Or

(a) Prove that the function defined as

$$f(x) = \begin{cases} x\left[\frac{e^{1/x}-1}{e^{1/x}+1}\right] & , x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is not derivable at $x = 0$.

Further show that f is continuous at $x = 0$.

(b) (i) If $z = x\tan^{-1}\left(x/y\right)$,

Show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$$

(ii) If $u = \log \frac{x^2+y^2}{x+y}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$$

Section IV

4. (a) Prove that the equation of the normal to the asteroid

$x^{2/3} + y^{2/3} = a^{2/3}$

may be written in the form

$x \sin\phi - y\cos\phi + a\cos 2\phi = 0.$

(b) Determine the position and nature of the double points on the curve

$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$

Or

(a) Trace the curve :

$x(x^2 + y^2) = a(x^2 - y^2)$

(b) Find the asymptotes of the curve

$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$

Section V

5. (a) State and prove Cauchy's Mean Value theorem.

(b) Find the extreme points of the function

$$f(x) = \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x,$$

where $x \in [0, \pi]$.

Or

(a) Prove that $x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}$

for all $x : 0 < x < 1$.

(b) Using Maclaurin's series expansions of $\log(1+x)$ and e^x, evaluate $\lim_{x \to 0}\left[\frac{(1+x)^{\frac{1}{x}} - e + \frac{ex}{2}}{x^2}\right]$

Section VI

6. (a) If $J_n = \int_{\pi/4}^{\pi/2} \cot^n \theta \, d\theta$ then show that

$n(J_{n+1} + J_{n-1}) = 1$. Hence deduce

that $\int_0^a \frac{x^7 dx}{(2a^2 - x^2)^4} = \frac{5}{12} - \frac{1}{2}\log 2.$

(b) Show that the length of a loop of the curve

$r^2 = a^2 \cos 2\theta$ is $2a \int_0^1 \frac{dt}{\sqrt{1-t^4}}$.

Or

(a) Evaluate

(i) $\int \frac{(2x+3)dx}{\sqrt{3+4x-4x^2}}$

(ii) $\int_0^\infty \frac{x^3 dx}{(1+x^2)^{9/2}}$

(b) Find the area of the surface generated by revolving the loop of the curve $9ay^2 = x(3a - x)^2$ about x-axis.