This question paper contains 5 printed pages.]

934 *Your Roll No.* ....

2009

B.A. Prog./I **I**

(R)

MATHEMATICS

Paper I

(Algebra and Calculus)

(New Course : Admissions of 2004 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note : *The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. 'A'). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.*

All sections are compulsory and have equal marks.

Attempt any ***two*** *parts from each section.*

[P.T.O.

Section I

1. (a) Do the vectors (1, 2, 1), (1, 0, –1) and (0, –3, 2) form a basis of $V = \mathbb{R}^3 (\mathbb{R})$. Give reasons. What is dim V ?

(b) Verify that the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix}$$

satisfies its characteristic equation. Hence compute A^4.

(c) Solve completely the system of equations

$$4x + 5y + 6z = 0$$
$$5x + 6y + 7z = 0$$
$$7x + 8y + 9z = 0$$

Section II

2. (a) If α, β are roots of the equation

$x^2 - 2x + 2 = 0$,

prove that :

$$\alpha^n + \beta^n = 2^{\left(\frac{n}{2}\right)+1} \cos\left(\frac{n\pi}{4}\right)$$

and hence evaluate $\alpha^6 + \beta^6$.

(b) Using De Moivre's theorem, solve the equation

$z^7 + z = 0$

(c) If the sum of two roots of the equation

$4x^4 - 24x^3 + 31x^2 + 6x - 8 = 0$

is zero, find all the roots of the equation.

Section III

3. (a) Show that function f defined as

$$f(x) = x \text{ when } 0 \le x < \frac{1}{2}$$
$$= 1 \text{ when } x = \frac{1}{2}$$
$$= 1 - x \text{ when } \frac{1}{2} < x < 1$$

is discontinuous at $x = \frac{1}{2}$. Examine the type of discontinuity.

(b) If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$,

find $\frac{d^2y}{dx^2}$ at $\theta = \pi$.

(c) If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, use Euler's theorem to prove $x\frac{\partial u}{\partial x}+y\frac{\partial u}{xy}=\frac{1}{4}\sin 2u$

Section IV

4. (a) Show that the pedal equation of the curve

$x = ae^{\theta}(\sin\theta - \cos\theta)$

$y = ae^{\theta}(\sin\theta + \cos\theta)$

is $r = \sqrt{2p}$

(b) Find the position and nature of the double points on the curve :

$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$

(c) Trace the curve :

$y^2(a^2 + x^2) = x^2(a^2 - x^2)$

Section V

5. (a) Explain why Roll's Theorem is not applicable to the function

$f(x) = 1 - x^{2/3}$ in $[-1, 1]$

(b) Find the values of a and b such that

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$$

(c) Show that

$x^5 - 5x^4 + 5x^3 - 1$

has a maximum value when $x = 1$, a minimum value when $x = 3$ and neither when $x = 0$.

Section VI

6. (a) Evaluate $\int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}$

(b) Find the surface of the solid generated by revolving the arc of the parabola $y^2 = 4ax$ bounded by its latus rectum about x-axis.

(c) Prove that the volume of the solid generated by the revolution of the curve $y^2 = \frac{a^3}{a^2+x^2}$ about its asymptote is $\frac{\pi^2 a^3}{2}$.