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Your Roll No.....

1144

2009

B.A. Prog/1

I-1

MATHEMATICS—Paper I

(Algebra and Calculus)

(NC : Admissions of 2006 onwards)

Time : 3 Hours

Maximum Marks : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Sections are compulsory and have equal marks.

Attempt any two parts from each Section.

Section I

1. (a) Prove that the following set of vectors in \mathbb{R}^3

$$S = \{(2, 0, -1), (5, 1, 0), (0, 1, 3)\}$$

is linearly independent.

P.T.O.

(b) If

$$A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix},$$

compute A^3 , A^4 and A^{-2} .

Or

(a) Verify that the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

satisfies its characteristic equation and hence

find A^{-1} .

(b) Solve by matrix method the following system of equations :

$$x + y + z = 7$$

$$x + 2y + 3z = 16$$

$$x + 3y + 4z = 22.$$

Section II

2. (a) Show that :

$$(1 + \cos \alpha + i \sin \alpha)^p + (1 + \cos \alpha - i \sin \alpha)^p$$

$$= 2^{p+1} \cos^p \left(\frac{\alpha}{2} \right) \cos \left(\frac{p\alpha}{2} \right).$$

(b) Find a necessary condition for the roots of the equation :

$$a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$$

to be in G.P. i.e. Geometric Progression.

Or

Prove that

(a) $32 \sin^4 \theta \cos^2 \theta = \cos 6\theta - 2 \cos 4\theta - \cos 2\theta + 2.$

(b) Solve the equation

$$16x^4 - 64x^3 + 56x^2 + 16x - 15 = 0.$$

Given that the roots are in Arithmetic Progression.

P.T.O.

Section III

3. (a) Discuss the derivability of the function f defined by

$$f(x) = \begin{cases} x & \text{for } x < 1 \\ 2 - x & \text{for } 1 \leq x \leq 2 \\ -2 + 3x - x^2 & \text{for } x > 2 \end{cases}$$

at $x = 1, 2$.

(b) If $u = e^{xyz}$, show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$$

Or

(a) Prove that the function

$$f(x) = \begin{cases} x \sin \frac{1}{x} & , \text{ if } x \neq 0 \\ 0 & , \text{ if } x = 0 \end{cases}$$

is continuous at $x = 0$ but not derivable at $x = 0$.

(b) If $x = a \sin^3 \theta, y = b \cos^3 \theta$,

find $\frac{d^2 y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

Section IV

4. (a) The tangents at two points P, Q of the cycloid :

$$x = a(\theta - \sin \theta), y = a(1 - \cos \theta)$$

are at right angles. Show that if P_1 and P_2 be the radii of curvatures at these points then :

$$P_1^2 + P_2^2 = 16a^2.$$

(b) Find the asymptotes of the curve :

$$(x^2 - y^2)(x + 2y) + 5(x^2 + y^2) + x + y = 0.$$

Or

(a) Trace the curve :

$$y^2(2a - x) = x^3.$$

(b) Show that the pedal equation of the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2 b^2}.$$

P.T.O.

Section V

5. (a) State and prove Rolle's Theorem. Show that there is no real number k for which the equation $x^2 - 3x + k = 0$ has two distinct roots in $[0, 1]$.
- (b) Obtain Maclaurin's series expansion of $\sin x$ for all $x \in \mathbb{R}$.

Or

- (a) Separate the interval in which the function :

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

is increasing or decreasing.

- (b) Evaluate any two of the following :

(i) $\lim_{x \rightarrow 0} (\cot x)^{1/\log x}$

(ii) $\lim_{x \rightarrow 0} \frac{1 + \sin x - \cos x + \log(1-x)}{x \tan^2 x}$

(iii) $\lim_{x \rightarrow \pi/2} (1 - \sin x) \tan x$

Section VI

6. (a) If

$$I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x \, dx,$$

prove that

$$I_{m,n} = \frac{n-1}{m+1} I_{m,n-2};$$

where m and n are positive integers.

- (b) Find the area enclosed by the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Or

- (a) Evaluate :

$$\int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}.$$

- (b) Find the volume of the solid obtained by the revolution of the loop of the curve $y^2(a+x) = x^2(a-x)$ about x -axis.