This question paper contains 7 printed pages]

Your Roll No.........................

1144

2009

B.A. Prog./I **I-1**

MATHEMATICS—Paper I

(Algebra and Calculus)

(NC : Admissions of 2006 onwards)

Time : 3 *Hours* *Maximum Marks* : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Sections are compulsory and have equal marks.

Attempt any *two* parts from each Section.

Section I

1. (*a*) Prove that the following set of vectors in $\mathbb{R}^3$

$$S = \{(2, 0, -1), (5, 1, 0), (0, 1, 3)\}$$

is linearly independent.

P.T.O.

(*b*) If

$$A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix},$$

compute A^3, A^4 and A^{-2}.

Or

(*a*) Verify that the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

satisfies its characteristic equation and hence find A^{-1}.

(*b*) Solve by matrix method the following system of equations :

$$x + y + z = 7$$

$$x + 2y + 3z = 16$$

$$x + 3y + 4z = 22.$$

Section II

2. (*a*) Show that :

$$(1 + \cos\alpha + i\sin\alpha)^p + (1 + \cos\alpha - i\sin\alpha)^p$$

$$= 2^{p+1}\cos^p\left(\frac{\alpha}{2}\right)\cos\left(\frac{p\alpha}{2}\right).$$

(*b*) Find a necessary condition for the roots of the equation :

$$a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$$

to be in G.P. i.e. Geometric Progression.

Or

Prove that

(*a*) $32\sin^4\theta\cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2.$

(*b*) Solve the equation

$$16x^4 - 64x^3 + 56x^2 + 16x - 15 = 0.$$

Given that the roots are in Arithmetic Progression.

Section III

3. (a) Discuss the derivability of the function f defined by

$$f(x)=\begin{cases} x & \text{for } x<1 \\ 2-x & \text{for } 1\le x\le 2 \\ -2+3x-x^2 & \text{for } x>2 \end{cases}$$

at $x = 1, 2$.

(b) If $u = e^{xyz}$, show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z}=\left(1+3xyz+x^2y^2z^2\right)e^{xyz}.$$

Or

(a) Prove that the function

$$f(x) = x \sin\frac{1}{x}, \text{ if } x \neq 0$$
$$= 0, \text{ if } x = 0$$

is continuous at $x = 0$ but not derivable at $x = 0$.

(b) If $x = a \sin^3 \theta$, $y = b \cos^3 \theta$,

find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

Section IV

4. (a) The tangents at two points P, Q of the cycloid :

$$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$$

are at right angles. Show that if P_1 and P_2 be the radii of curvatures at these points then :

$$P_1^2 + P_2^2 = 16a^2.$$

(b) Find the asymptotes of the curve :

$$\left(x^2-y^2\right)(x+2y)+5\left(x^2+y^2\right)+x+y=0.$$

Or

(a) Trace the curve :

$$y^2(2a - x) = x^3.$$

(b) Show that the pedal equation of the ellipse :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

is

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}-\frac{r^2}{a^2b^2}.$$

P.T.O.

Section V

5. (a) State the prove Rolle's Theorem. Show that there is no real number k for which the equation $x^2 - 3x + k = 0$ has two distinct roots in [0, 1].

(b) Obtain Maclaurin's series expansion of $\sin x$ for all $x \in \mathbf{R}$.

Or

(a) Separate the interval in which the function :

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

is increasing or decreasing.

(b) Evaluate any *two* of the following :

(i) $\lim_{x \to 0} (\cot x)^{1/\log x}$

(ii) $\lim_{x \to 0} \frac{1 + \sin x - \cos x + \log(1 - x)}{x \tan^2 x}$

(iii) $\lim_{x \to \pi/2} (1 - \sin x) \tan x$.

Section VI

6. (a) If

$$I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x \, dx,$$

prove that

$$I_{m,n} = \frac{n-1}{m+1} I_{m,n-2};$$

where m and n are positive integers.

(b) Find the area enclosed by the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Or

(a) Evaluate :

$$\int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}.$$

(b) Find the volume of the solid obtained by the revolution of the loop of the curve $y^2(a + x) = x^2(a - x)$ about x-axis.