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Your Roll No............................

1144

2009

B.A. Prog./I   **I-1**

MATHEMATICS—Paper I

(Algebra and Calculus)

(NC : Admissions of 2006 onwards)

Time : 3 Hours   *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Sections are compulsory and have equal marks.

Attempt any *two* parts from each Section.

Section I

1. (a) Prove that the following set of vectors in $\mathbf{R}^3$

$$S = \{(2, 0, -1), (5, 1, 0), (0, 1, 3)\}$$

is linearly independent.

P.T.O.

(b) If

$$A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix},$$

compute A^3, A^4 and A^{-2}.

Or

(a) Verify that the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

satisfies its characteristic equation and hence find A^{-1}.

(b) Solve by matrix method the following system of equations :

$$x + y + z = 7$$

$$x + 2y + 3z = 16$$

$$x + 3y + 4z = 22.$$

Section II

2. (a) Show that :

$$(1 + \cos \alpha + i \sin \alpha)^p + (1 + \cos \alpha - i \sin \alpha)^p$$

$$= 2^{p+1} \cos^p \left(\frac{\alpha}{2} \right) \cos \left(\frac{p\alpha}{2} \right).$$

(b) Find a necessary condition for the roots of the equation :

$$a_0 x^3 + 3a_1 x^2 + 3a_2 x + a_3 = 0$$

to be in G.P. i.e. Geometric Progression.

Or

Prove that

(a) $32 \sin^4 \theta \cos^2 \theta = \cos 6\theta - 2 \cos 4\theta - \cos 2\theta + 2.$

(b) Solve the equation

$$16x^4 - 64x^3 + 56x^2 + 16x - 15 = 0.$$

Given that the roots are in Arithmetic Progression.

P.T.O.

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Section III

3. (a) Discuss the derivability of the function f defined by

$$f(x) = \begin{cases} x & \text{for} \quad x < 1 \\ 2 - x & \text{for} \quad 1 \le x \le 2 \\ -2 + 3x - x^2 & \text{for} \quad x > 2 \end{cases}$$

at $x = 1,\ 2$.

(b) If $u = e^{xyz}$, show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = \left(1 + 3xyz + x^2 y^2 z^2\right) e^{xyz}.$$

Or

(a) Prove that the function

$$\begin{aligned} f(x) &= x \sin \frac{1}{x} \quad , \quad \text{if } x \ne 0 \\ &= 0 \quad , \quad \text{if } x = 0 \end{aligned}$$

is continuous at $x = 0$ but not derivable at $x = 0$.

(b) If $x = a \sin^3 \theta,\ y = b \cos^3 \theta$, find $\dfrac{d^2 y}{dx^2}$ at $\theta = \dfrac{\pi}{4}$.

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Section IV

4. (a) The tangents at two points P, Q of the cycloid :

$$x = a\ (\theta - \sin\ \theta),\ y = a\ (1 - \cos\ \theta)$$

are at right angles. Show that if P_1 and P_2 be the radii of curvatures at these points then :

$$P_1^2 + P_2^2 = 16a^2.$$

(b) Find the asymptotes of the curve :

$$\left(x^2 - y^2\right)(x + 2y) + 5\left(x^2 + y^2\right) + x + y = 0.$$

Or

(a) Trace the curve :

$$y^2\ (2a - x) = x^3.$$

(b) Show that the pedal equation of the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2 b^2}.$$

P.T.O.

Section V

5. (a) State the prove Rolle's Theorem. Show that there is no real number k for which the equation $x^2 - 3x + k = 0$ has two distinct roots in [0, 1].

   (b) Obtain Maclaurin's series expansion of $\sin x$ for all $x \in \mathbf{R}$.

   Or

   (a) Separate the interval in which the function :

   $$f(x) = 2x^3 - 15x^2 + 36x + 1$$

   is increasing or decreasing.

   (b) Evaluate any *two* of the following :

   (i) $\displaystyle \lim_{x \to 0} (\cot x)^{1/\log x}$

   (ii) $\displaystyle \lim_{x \to 0} \frac{1 + \sin x - \cos x + \log(1 - x)}{x \tan^2 x}$

   (iii) $\displaystyle \lim_{x \to \pi/2} (1 - \sin x) \tan x$

Section VI

6. (a) If

   $$I_{m, n} = \int_{0}^{\pi/2} \sin^m x \cos^n x \, dx,$$

   prove that

   $$I_{m, n} = \frac{n - 1}{m + 1} I_{m, n - 2};$$

   where m and n are positive integers.

   (b) Find the area enclosed by the ellipse :

   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

   Or

   (a) Evaluate :

   $$\int_{0}^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}.$$

   (b) Find the volume of the solid obtained by the revolution of the loop of the curve $y^2(a + x) = x^2(a - x)$ about x-axis.