Reg.No										

MANIPAL INSTITUTE OF TECHNOLOGY

(A constituent college of Manipal University, Manipal -576

FIRST SEMESTER BE DEGREE END SEMESTER EXAMINATIONS – 2007

SUB: ENGG.MATHEMATICS – I (MAT – 101)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : Answer any FIVE full questions.**

1A. Find the n^{th} derivative of the following

(i) $\dfrac{x^2}{2x^2 + 7x + 6}$ (ii) coshx. cos3x

1B. Find the evolute of the curve $y^2 = 4ax$.

1C. Find the image of the line $\dfrac{x-1}{2} = \dfrac{y-2}{1} = \dfrac{z-3}{4}$ in the plane $2x + y + z = 6$.

(4 + 3 + 3)

2A. If $y = \tan^{-1} x$, show that

$(1 + x^2)y_{n+2} + 2(n+1)\, xy_{n+1} + n\,(n+1)\, y_n = 0$

2B. Find the circle of curvature at the point (0, 1) on the curve $y = x^3 + 2x^2 + x + 1$.

2C. Find the length of the parabola $y^2 = 4ax$ from the vertex to one extremity of the latus rectum

(3 + 3 + 4)

3A. Find the angle between two curves (in its simplest form) at the point of intersection given $r^2 = a^2 \cos 2\theta$, $r = a(1 + \cos\theta)$.

3B. (i) State integral test

(ii) Test the nature of $\sum_{1}^{\infty} \sqrt{\dfrac{n}{n+1}}\, x^n, \quad x > 0$

3C. Find the percentage error in r if 2% error is made in measurement of r_1 and r_2 given $\dfrac{1}{r} = \dfrac{1}{r_1} + \dfrac{1}{r_2}$. (4 + 3 + 3)

P.T.O

4A. Sketch and find the area of the loop of the curve $y^2 (a + x) = x^2 (a – x)$

4B. Find first three non zero terms in Maclurin's series expansion of $f(x) = \tan x$

4C. (i) State Cacuhy's root test.
(ii) Test the convergence of

$$1 + \frac{a}{b} + \frac{a(a+1)}{b(b+1)} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)} + \ldots \qquad \text{(if } a,\ b > 0.\text{)}$$

(3 + 3 + 4)

5A. (j) Obtain the reduction formula for $\int \cos^n x\, dx$ hence evaluate

$$\int_0^{\pi/2} \sin^n x\, dx$$

(ii) $$\int_0^1 \frac{x^9}{\sqrt{1-x^2}}\, dx$$

5B. (i) State Lagrange's mean value theorem.

(ii) Verify Cauchy's mean value theorem for $f(x) = \log_e x$, $g(x) = \frac{1}{x}$ in $[1, e]$

5C. Trace with explanation $r^2 = a^2 \cos 2\theta$ and find its area.

(4 + 3 + 3)

6A. Find the equation of the right circular cone generated when the straight line $2y + 3z = 6$, $x = 0$ revolves about z – axis.

6B. Evaluate :

(i) $$\lim_{x \to 0} \left(\frac{1}{x} - \cot x \right)$$

(ii) $$\lim_{x \to 0} (\sin x)^{\tan x}$$

6C. Find the points on the lines

$$\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$$

$$\frac{x}{(-3)} = \frac{y+9}{2} = \frac{z-2}{4}$$

which are nearest to each other. Hence find the shortest distance between the lines. (3 + 4 + 3)

****

MANIPAL INSTITUTE OF TECHNOLOGY

(A constituent college of Manipal University, Manipal –576

FIRST SEMESTER BE DEGREE END SEMESTER EXAMINATIONS – 2007

SUB: ENGG.MATHEMATICS – I (MAT – 101)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : Answer any FIVE full questions.**

1A. Find the n^{th} derivative of

(i) $\frac{4x}{(x-1)^2(x+1)}$ (ii) sinx. Sin2x.sin3x

1B. Find the evolute of $x^{2/3} + y^{2/3} = a^{2/3}$, $(a > 0)$.

1C. Find the image of the point (1, 2, 3) in the plane x + 2y + 3z = 21.

(4 + 3+ 3)

2A. If $y^{1/m} + y^{-1/m} = 2x$, prove that
$(x^2 - 1)\, y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

2B. If ρ be the radius of curvature at any point P on the parabola $y^2 = 4ax$ and S be its focus, then show that ρ^2 varies as $(SP)^3$.

2C. Find the volume of the solid formed by revolving the curve $y^2(2a - x) = x^3$ about its asymptote.

(4 + 3 + 3)

3A. Find the angle of intersection of the curves $r = \frac{a}{1+\cos\theta}$ and $r = \frac{b}{1-\cos\theta}$ (in its Simplest form).

3B. Define :
(i) Absolute convergence
(ii) Conditionally convergence
Find the nature of the series

$$\frac{1}{2}x + \frac{1.3}{2.4}x^2 + \frac{1.3.5}{2.4.6}x^3 + \ldots$$

P.T.O

3C. The pressure p and the volume v of a gas are connected by $pv^{1.4} = K$. Find the percentage increase in the pressure corresponding to a diminution of $\frac{1}{2}\%$ in the volume, if K is constant.

(4 + 3+ 3)

4A. Sketch and find the area enclosed by the curve $a^2y^2 = x^3(2a - x)$.

4B. Expand tanx in powers of $\left(x - \frac{\pi}{4}\right)$ up to three terms.

4C. State D'Alembert's ratio test.
Test the nature of the series

$$\frac{1}{4.7.10} + \frac{4}{7.10.13} + \frac{9}{10.13.16} + \frac{16}{13.16.19} + \ldots$$

(3 + 3+ 4)

5A. (i) Obtain the reduction formula for $\int \sin^n x \, dx$

(iii) Evaluate : $\int_0^{2a} \frac{x^3}{\sqrt{(2ax - x^2)}} dx$

5B. (i) State Rolle's theorem.
(ii) Verify Lagrange's mean value theorem for $f(x) = \log x$ in $[e, e^2]$

5C. Trace and find the length of one arch of the cycloid $x = a(\theta - \sin\theta)$;
$y = a(1 - \cos\theta)$.

(4 + 3+ 3)

6A. The radius of a normal section of a right circular cylinder is 2 units; the axis lies along the straight line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$, find its equation.

6B. Evaluate the following :

(i) $\lim_{x \to \pi/4} (\tan x)^{\tan 2x}$ (ii) $\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{x \tan x}\right]$

6C. Show that the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and
$3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$ are coplanar. Find their point of intersection and the plane in which they lie.

(4 + 3+ 3)

****