Math 1.3

I Semester M.Sc. Examination, May 2011
MATHEMATICS
Complex Analysis – I

Time : 3 Hours	Max. Marks : 80

***Instructions** : Answer **any five** questions. **Each** question carry **equal** marks.*

1. a) For each positive integer n, show that

$$1 + z + z^2 + \ldots + z^{n-1} = \frac{1 - z^n}{1 - z}.$$ **(4+4+8)**

b) State and prove triangular inequality.

c) Prove that $\left|\frac{z_1 - z_2}{1 - \bar{z}_1 z_2}\right| = 1$ if either $|z_1| = 1$ or $|z_2| = 1$. What exception must be mode if $|z_1| = |z_2| = 1$?

2. a) If Re z > 0, then prove that $\mathrm{Re}\left(\bar{z}\sqrt{z^2 - 1}\right) \geq 0$. **(8+8)**

b) Show that z_1 and z_2 corresponds to diametrically opposite points on the Riemann sphere if and only if $z_1\bar{z}_2 = -1$.

3. a) Find the general equation of a straight line. **(6+4+6)**

b) Show that if the equation $z^2 + \alpha z + \beta = 0$ has a pair of conjugate complex roots, then α, β are both real and $\alpha^2 < 4\beta$.

c) Define a continuity of a function. Prove that f(z) = u(x, y) + iv(x, y) on continuous at $z_0 = x_0 + iy_0$ if and only if u and v are continuous at (x_0, y_0).

P.T.O.

4. a) Deduce the polar form of a Cauchy-Riemann equations. **(6+4+6)**

   b) Show that $f(z) = \sqrt{r}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)$. Where $r > 0$ and $0 < \theta < 2\pi$ is differentiable and find $f'(z)$.

   c) State and prove necessary and sufficient condition for a required to be convergent.

5. a) State and prove Weierstrass m-test. **(6+6+4)**

   b) Prove that the power series $\sum_{n=0}^{\infty} na_n z^{n-1}$ obtained by differentially the power series $\sum_{n=0}^{\infty} a_n z^n$ has same radius of convergence of as the original series $\sum_{n=0}^{\infty} a_n z^n$.

   c) Find the region of convergence of the series $\sum_{n=0}^{\infty} \left(1+\frac{1}{n}\right)^{n^2} z^n$.

6. a) Prove that bilinear transformation preserves cross ratio. **(6+6+4)**

   b) Let f(z) be a function which is continuous on any continuous rectifiable curve C and W be any point of the complex place not lying in C. Then prove that $F(w) = \int_c \frac{f(z)}{z-w} dz$ is differentiable.

   c) Evaluate the integral $\int_c x^2 - iy^2 \, dz$ where C is the parabola $y = 2x^2$ from (1, 2) to (2, 8).

7. a) State and prove Cauchy theorem for a disk. **(8+8)**

   b) State and prove Cauchy integral formula.

8. a) State and prove Liouville's theorem and hence deduce fundamental theorem of Algebra. **(8+8)**

   b) State and prove Taylor's theorem.

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