Math 1.5

I Semester M.Sc. Examination, May 2011
MATHEMATICS
Differential Equations

Time : 3 Hours Max. Marks : 80

Note : *1) Answer **any five** questions.*
*2) **All** questions carry **equal** marks.*

1. a) State and prove Picard's theorem for existence and uniqueness of an initial value problem. **12**

   b) Find the general solution of $y^{(5)} - y^{(4)} - y' + y = 0$ **4**

2. a) Find the general solution of $y'' + y = \tan x \sec x$ by the method of variation of parameters. **8**

   b) Find the solution of $x^2 y'' + 7xy' + 8y = 0$ by finding the solution of its adjoint equation by stating appropriate result. **8**

3. a) Define Strum-Limville Problem. Find the eigen values and eigen function of Strum-Limville Problem. **8**

   b) Solve the boundary value problem $y'' + \lambda y = f(x)$, $y(0) = 0$, $y(\pi) = 0$ by Green's function method. **8**

4. a) Find the series solution of laguerre differential equation $xy'' + (1 - x)y' + ny = 0$ around the singular point for a non-negative positive integer n. **9**

   b) Prove that $\int_0^\infty e^{-x} L_m(x) L_n(x)\, dx = \begin{cases} 1 \text{ if } m = n \\ 0 \text{ if } m \neq n \end{cases}$ **7**

P.T.O.

5. a) Find the general solution of the non-homogeneous linear system

$$X'(t) = \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix} X(t) + \begin{bmatrix} 3t \\ e^{-t} \end{bmatrix}$$ **6**

b) Define the four types of critical points of the linear autonomous system

$$\frac{dx}{dt} = ax + by$$

$$\frac{dy}{dt} = cx + dy, \text{ when } ad - bc \neq 0.$$ **6**

c) Determine the nature and stability of critical point of the system

$$\frac{dx}{dt} = -2x + 4y \; ; \; \frac{dy}{dt} = -2x + by.$$ **4**

6. a) Find the critical points of the non-linear system $\frac{dx}{dt} = 1 - y : \frac{dy}{dt} = x^2 - y^2$. Determine their nature and stability. **8**

b) State only the Liapunov stability theorem. Determine the stability of the critical point (0, 0) of the system $\frac{dx}{dt} = -x^5 - y^3 \; ; \; \frac{dy}{dt} = 3x^3 - y^3$. **8**

7. a) State Cauchy problem and discuss its geometrical significance. **8**

b) Define semilinear, quasilinear and nonlinear partial differential equations of first order. Explain the method of characteristics for solving quasilinear cauchy problem. **8**

8. Solve :

a) $$\frac{\partial^3 z}{\partial x^3} + \frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} - \frac{\partial^3 z}{\partial y^3} = e^{2x+y} + \sin(x+2y).$$ **6**

b) Discuss the method of characteristics for the cauchy problem of second order and illustrate with a suitable example. **10**
