

Math 1.1

I Semester M.Sc. Mathematics Examination, May 2011 ALGEBRA

Time : 3 Hours

Max. Marks : 80

Note : 1) Answer **any five** questions.
2) **All** questions carry **equal** marks.

1. a) State and prove Lagrange theorem for finite groups.
b) Let $f : G \rightarrow G'$ be a group homomorphism. Then prove that $\ker f$ is a normal subgroup of G . Moreover prove that f is a one-one mapping if and only if $\ker f = \{e\}$. **(8+8)**
 2. a) Prove that every permutation $\sigma \in S_n$ can be expressed as a product of disjoint cycles.
b) Prove the class equation of the group G . **(8+8)**
 3. a) State and prove the first Sylow theorem.
b) Show that any group of order $5^2 \cdot 7^2$ is abelian. **(8+8)**
 4. a) Show that any integral domain can be embedded in a field.
b) Let R be a commutative ring with identity. Then prove that R is a field if and only if the only ideals of R are $\{0\}$ and R itself. **(10+6)**
 5. a) State and prove the fundamental theorem of homomorphism for rings.
b) Let R be a commutative ring with identity. Prove that an ideal P in R is a prime ideal if and only if R/P is an integral domain. **(8+8)**
 6. a) Prove that in a Unique factorization domain, an element is a prime if and only if it is irreducible.
b) Let F be a field and $f(x) \in F[x]$. Then prove that $\alpha \in F$ is a root of $f(x)$ if and only if $(x - \alpha)$ divides $f(x)$. **(8+8)**
 7. a) Let W be a subspace of a finite-dimensional vector space V . Then prove that W is finite - dimensional and $\dim W \leq \dim V$.
b) If $F \subseteq K \subseteq L$ are fields, then prove that $[L : F] = [L : K] [K : F]$. **(8+8)**
 8. a) Prove that any splitting field of a polynomial over F is a normal extension of F .
b) State and prove the Primitive Element Theorem. **(8+8)**
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