BTC – 31

III Semester B.Tech. Examination, Feb./March 2010
ENGINEERING MATHEMATICS – 3 (Discrete Maths)

Time : 3 Hours | Max. Marks : 80

Instructions : *1) Answer **all** questions in Part **A**, **6** out of 8 questions in Part **B** and **3** out of 5 questions in Part **C**.*
*2) Part **A** : Questions from **1** to **8** carry **1** mark **each** and **9** to **14** carry **2** marks **each**.*
*3) Part **B** : **Each** question carries **5** marks.*
*4) Part **C** : **Each** question carries **10** marks.*

PART – A

1. Define Union and Intersection of two sets A and B.

2. Define Power set with an example.

3. Let A = {1, 2, 3} and B = {2, 4, 5}. Determine the number of relations from A to B.

4. Define a function. Give an example.

5. Let p, q, r be propositions having truth values T, F, F respectively. Find the truth value of $(p \vee q) \vee r$.

6. State the converse of – if a quadrilateral is a parallelogram, then its diagonals bisect each other.

7. Define the sum rule.

8. How many different signals can be made by 5 flags from 8 flags of different colors?

9. Determine the sets A and B, given that A – B = {1, 3, 7, 11} B – A= {2, 6, 8} and $A \cap B$ = {4, 9}.

10. Let A = {1, 2, 3} and B = {2, 4, 5}.Determine the number of relations from A, B that contain exactly five ordered pairs.

P.T.O.

11. Let A = {0, ±1, ±2, ±3}. Consider the function f: $A \rightarrow R$ (Where R is the set of real numbers) defined by $f(x) = x^3 - 2x^2 + 3x + 1$, for $x \in A$. Find the range of f.

12. Let p and q be primitive statements for which the implication $p \rightarrow q$ is false. Determine the truth value of $p \wedge q$.

13. How many numbers of three distinct digits can be formed from 1, 2, 3, 4, 5?

14. Define: (i) Simple graph (ii) Multi graph

    Give one example for each.

PART – B

1. For any two sets A and B, prove that
   i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

   ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

2. Define the following relations:
   i) Reflexive ii) Symmetric iii) Anti-Symmetric

   Give one example for each.

3. Define a Tautology. Prove that the following compound proposition is a tautology.

   $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow p \rightarrow r$

4. State the laws of Boolean algebra.

5. In any undirected graph, prove that the number of odd degree vertices is even.

6. Prove by mathematical induction that, for all positive integers $n \geq 1$.

   $1 + 2 + 3 + \text{- - - - - - - - - - - - - -} + n = \frac{1}{2} n (n+1)$.

7. Let G be the set of all non-zero real numbers and let a*b=1/2ab. Show that (G,*) is an abelian group.

8. Define Homomorphism, and Isomorphism of groups.
   Define f: $R \rightarrow R^+$ by $f(x) = \mathbf{e}^x$ for all $x \in R$. Verify that f is an isomorphism.

PART – C

1. Using Venn diagram, prove that, for any three sets A, B, C

   $A \Delta (B \Delta C) = (A \Delta B) \Delta C$

2. For any propositions p,q,r, prove the following:

   i) $[(p \rightarrow q) \wedge (p \rightarrow r)] \leftrightarrow (q \wedge r)$

   ii) $[(p \rightarrow q) \wedge (r \rightarrow q)] \leftrightarrow [(p \vee r) \rightarrow q]$

3. Let A = {1, 2, 3, 4, 5}. Define a relation R on A × A by (x_1, y_1) R (x_2, y_2) if and only if $x_1 + y_1 = x_2 + y_2$.

   i) Verify that R is an equivalence relation on A × A.

   ii) Determine the equivalence classes [(1, 3)], [(2, 4)] and [(1, 1)].

4. Define Euler graph. A given connected graph G is an Euler graph if all vertices of G are of even degree.

5. Prove that the intersection of two subgroups of a group is a subgroup of the group. Is the union of two subgroups of the group a subgroup of the group? Justify your answer.